Visualizing PML

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The PML Visualization Project

dumas.io/PML

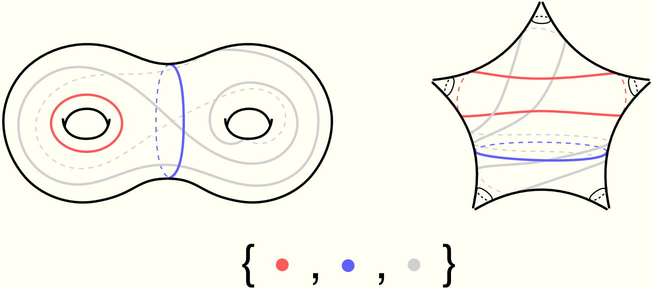
Joint work with François Guéritaud (Univ. Lille)

I will also demonstrate 3D graphics software developed by UIC undergraduate researchers Galen Ballew and Alexander Gilbert.

What is PML?

The space of Projective Measured Laminations

- A completion of the set $\mathcal{C}$ of simple closed curves on S
- Homeomorphic to $\mathbf{S}^{N-1}$, where $N = \dim(\mathcal{T})$
- Piecewise linear structure, PL action of Mod(S)



Linear analogy

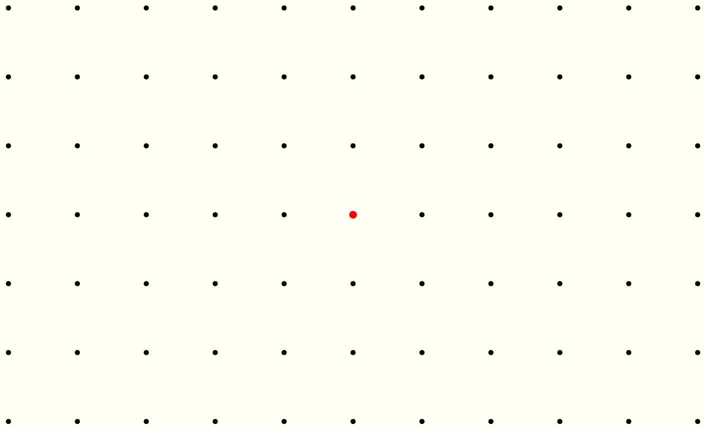
The inclusions

$$\mathcal{C} \hookrightarrow \mathrm{ML} \qquad \text{(discrete image)}$$
$$\mathcal{C} \hookrightarrow \mathrm{PML} \qquad \text{(dense image)}$$

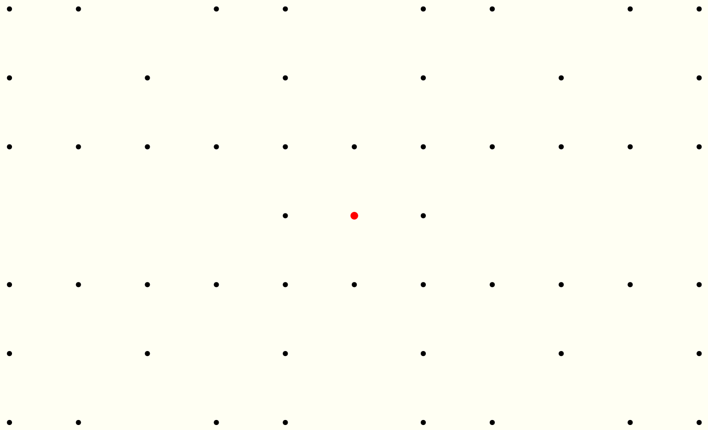
are analogous to

$$\mathrm{primitive}(\mathbf{Z}^N) \hookrightarrow \mathbf{R}^N \qquad \text{(discrete image)}$$
$$\mathrm{primitive}(\mathbf{Z}^N) \hookrightarrow \mathbf{S}^{N-1} \qquad \text{(dense image)}$$

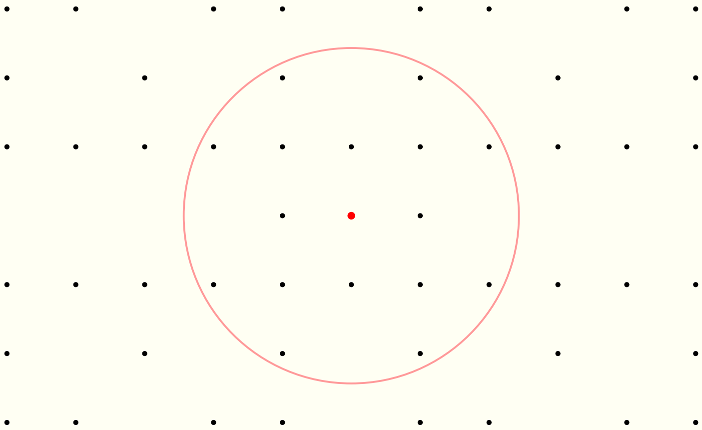
Linear visualization



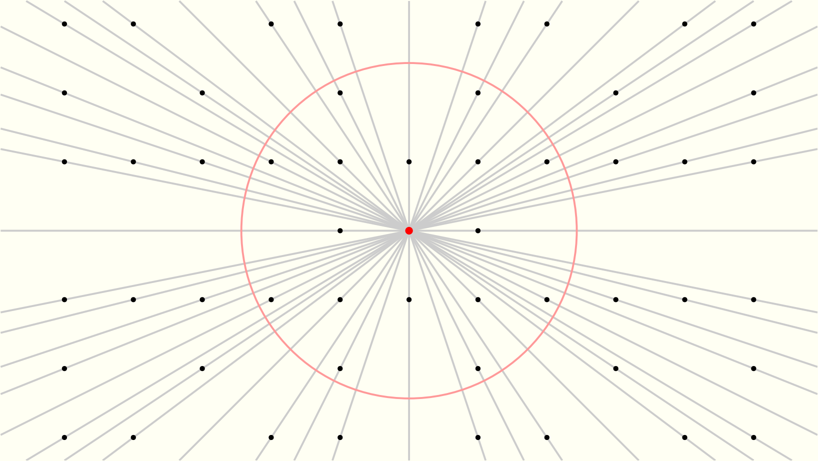
Linear visualization



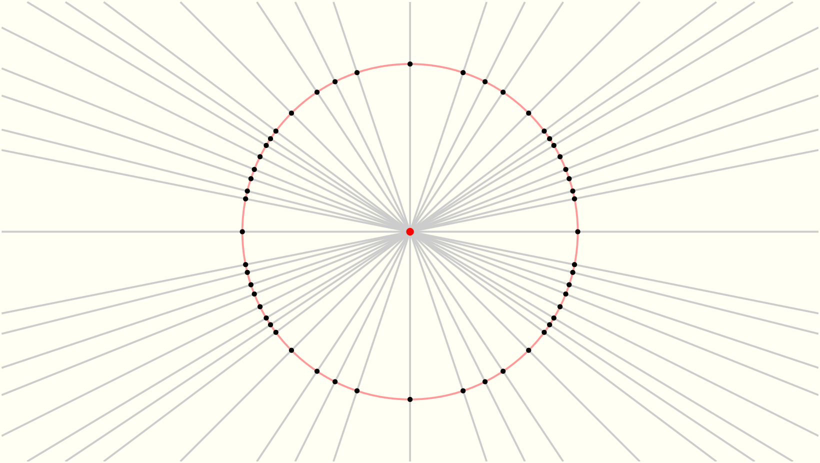
Linear visualization



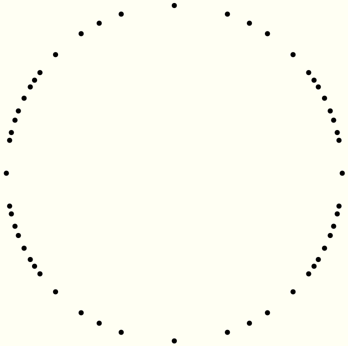
Linear visualization



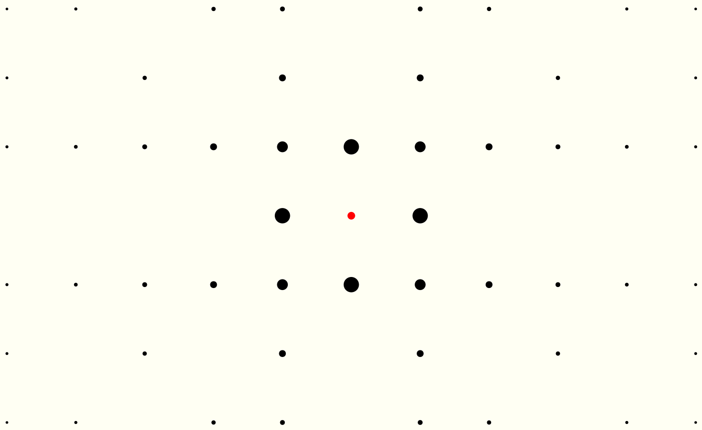
Linear visualization



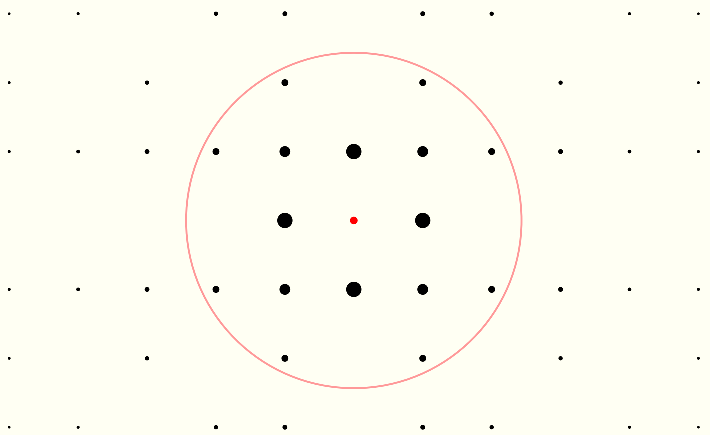
Linear visualization



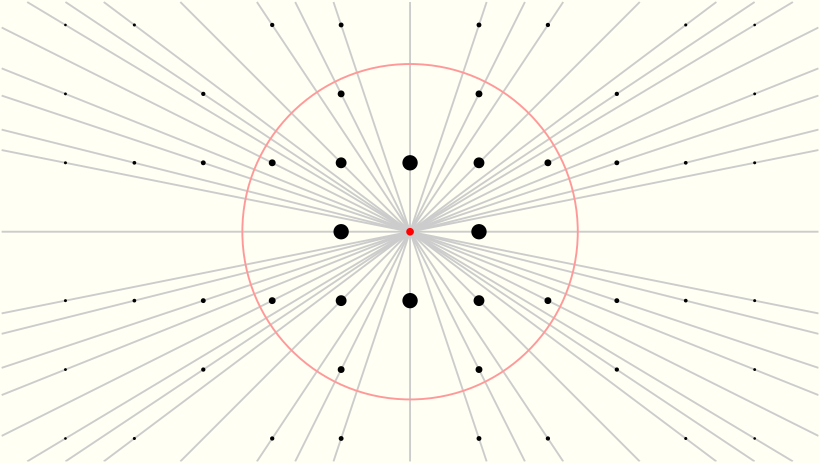
Linear visualization



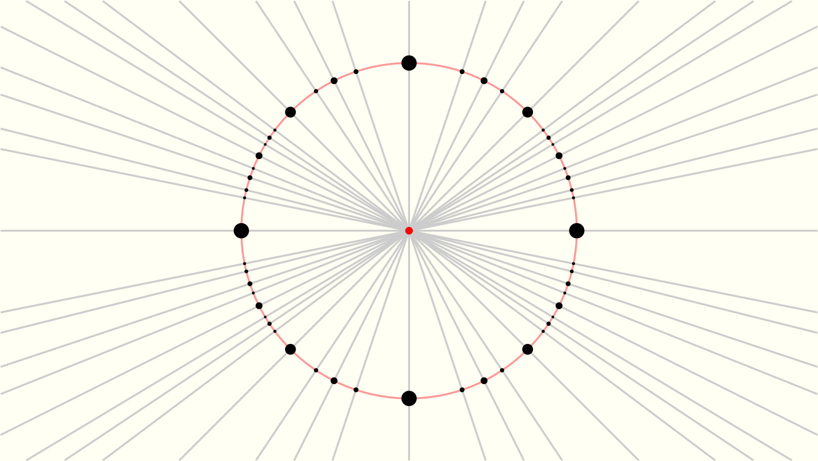
Linear visualization



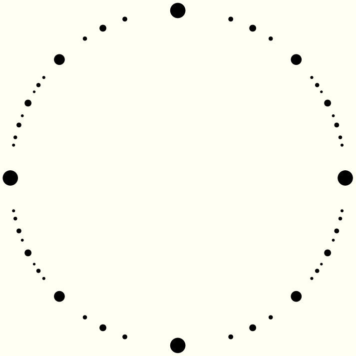
Linear visualization



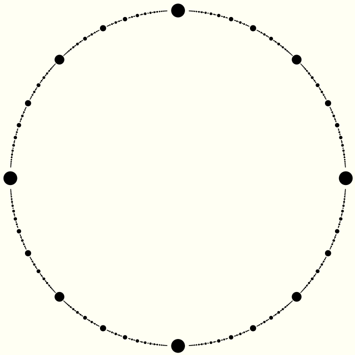
Linear visualization



Linear visualization



Linear visualization



Not so fast

Can we visualize PML similarly?

Several issues:

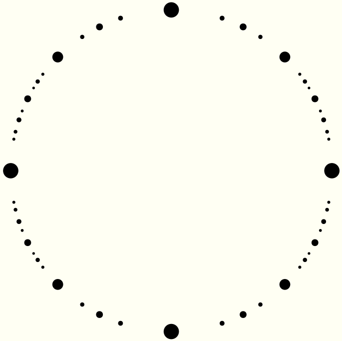
- Need to choose an identification ML $\simeq \mathbf{R}^N$.
  (Train tracks? Dehn-Thurston? Something else?)

- The “small” values of $N = 6g - 6 + 2n$ are

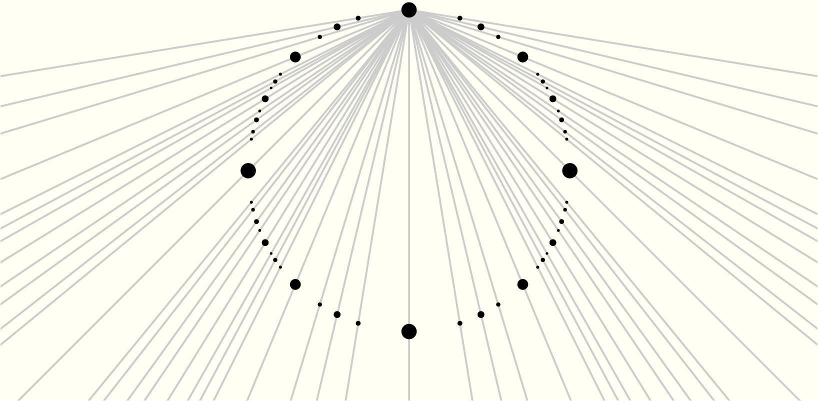
  N=2 for $S_{0,4}$ and $S_{1,1}$

  N=4 for $S_{0,5}$ and $S_{1,2}$

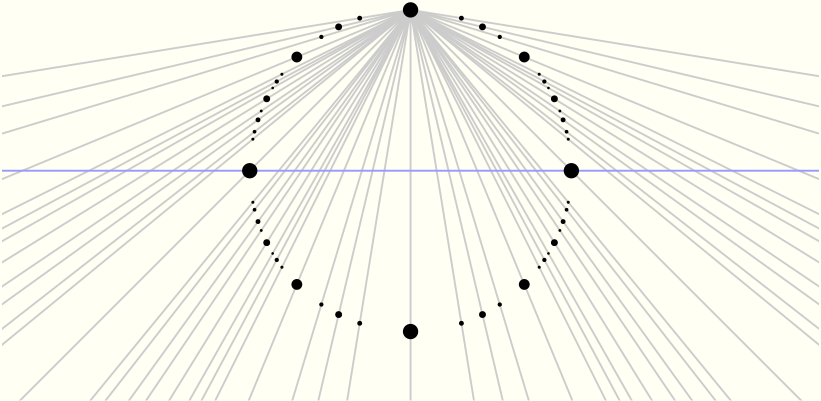
Stereographic projection



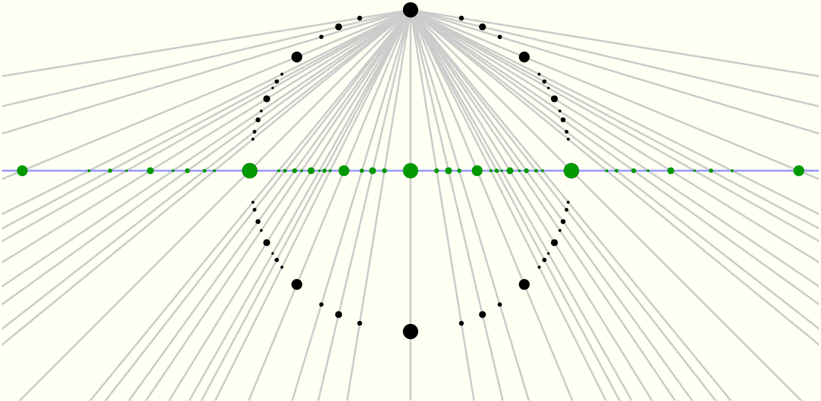
Stereographic projection



Stereographic projection



Stereographic projection



Stereographic projection

Stereographic projection

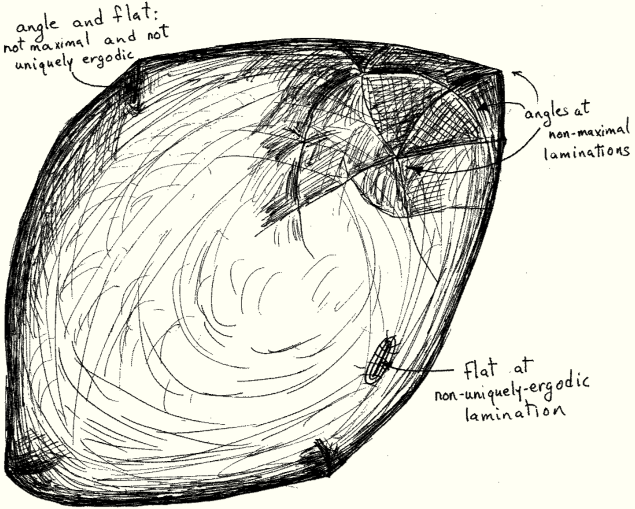
Thurston's embedding

Fix $X \in \mathcal{T}(S)$, the base hyperbolic structure.

$$\begin{aligned} \mathrm{PML} &\to T^*_X\mathcal{T}(S) \\ [\lambda] &\mapsto d_X \log(\ell_\lambda) \end{aligned}$$

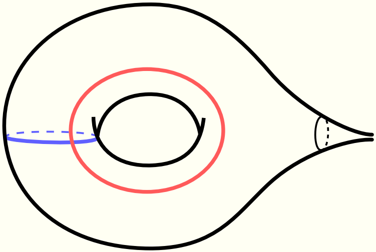
Curve $\alpha \in \mathcal{C}$ maps to a vector representing the sensitivity of its geodesic length to deformations of the hyperbolic structure X.

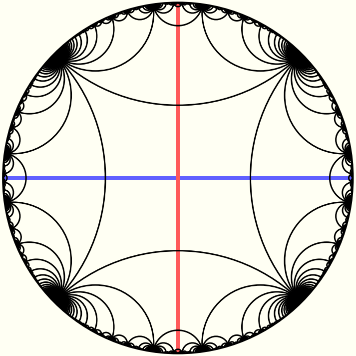
Thurston’s drawing of PML



From “Minimal stretch maps between hyperbolic surfaces”, preprint, 1986.

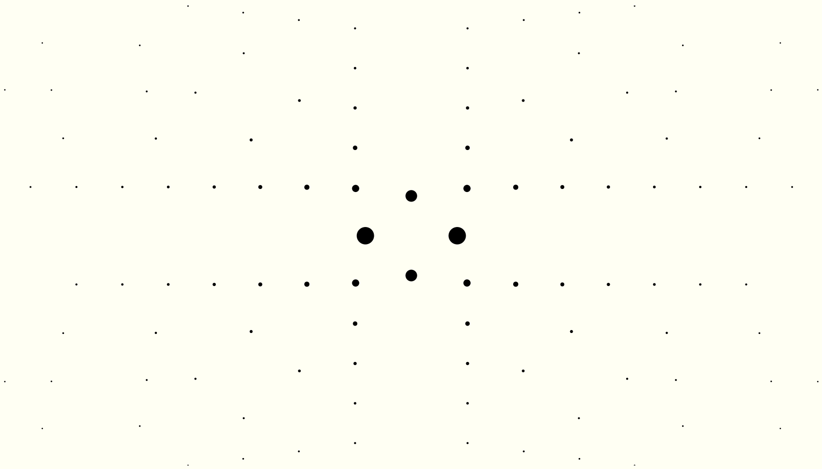
Punctured torus



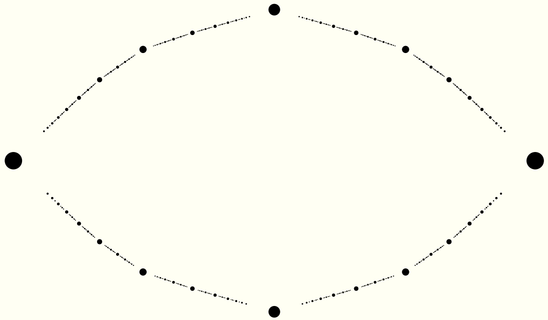


$S_{1,1}$

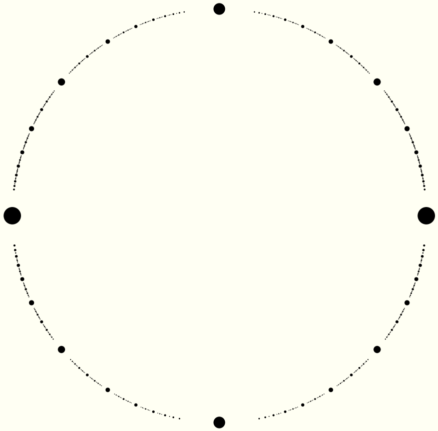
Punctured torus



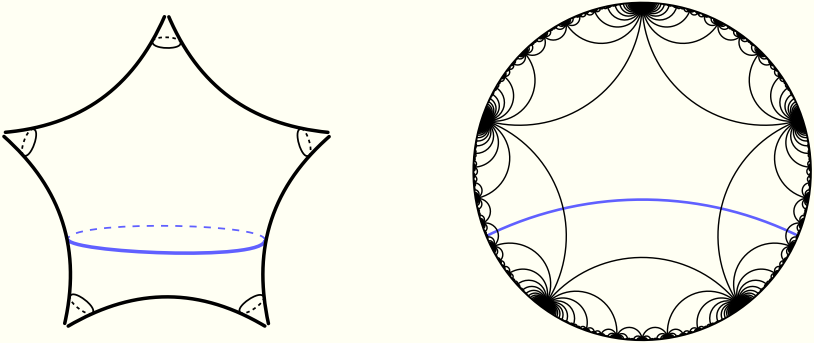
Punctured torus



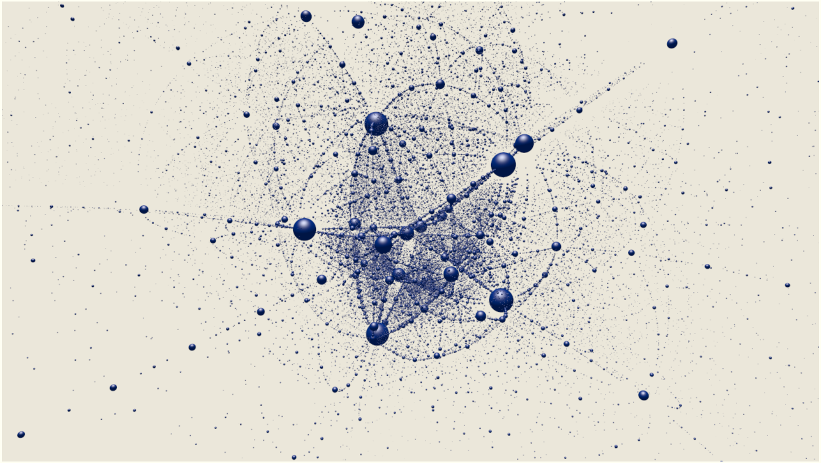
Punctured torus



Five-punctured sphere

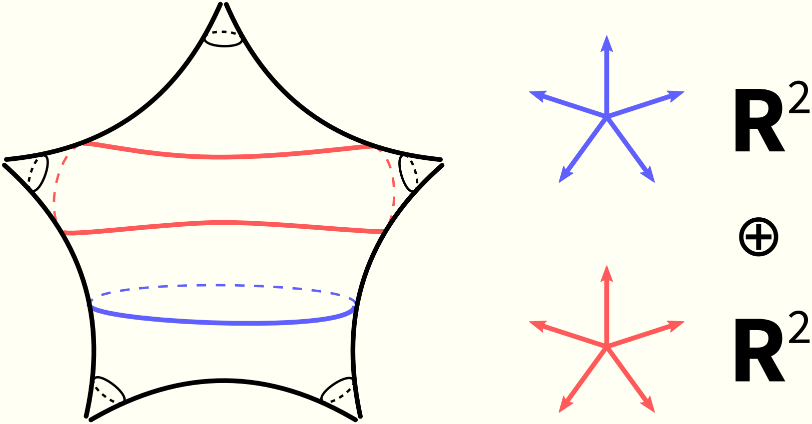


$S_{0,5}$

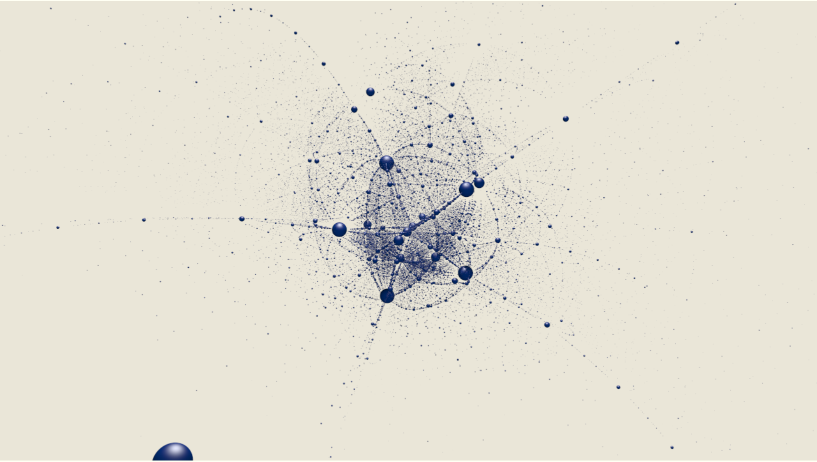


pmls05-001

Earthquake basis

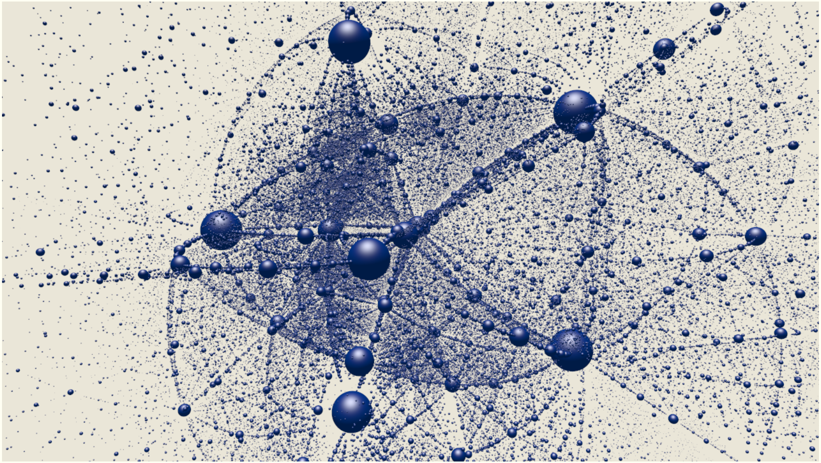


Rotating the pole



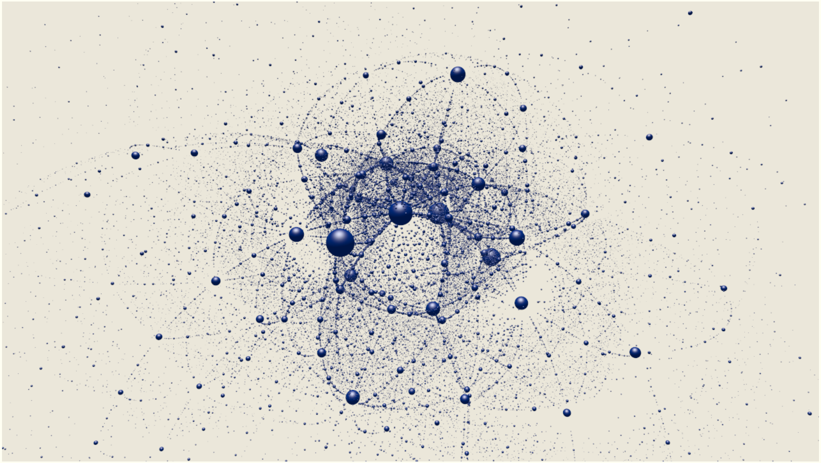
pmls05-010

Closer?



pmls05-020

Clifford flow



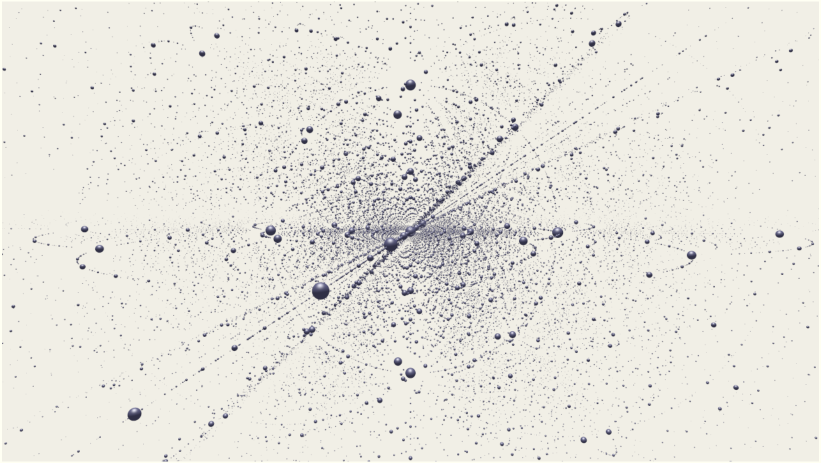
pmls05-030

Back to the linear analogy

It is “easy” to imagine $\mathbf{Z}^4$.

What about its stereographic projection?

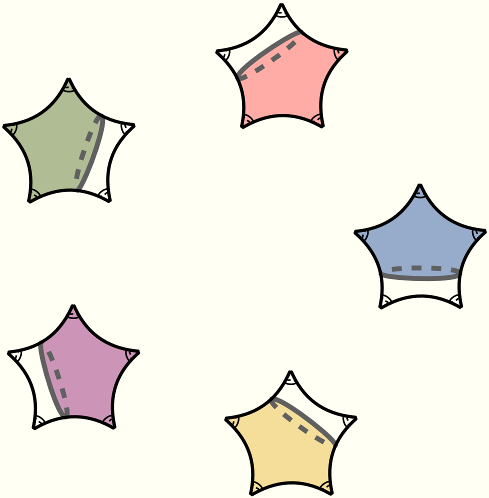
And can this inform our understanding of the PML($S_{0,5}$) images?



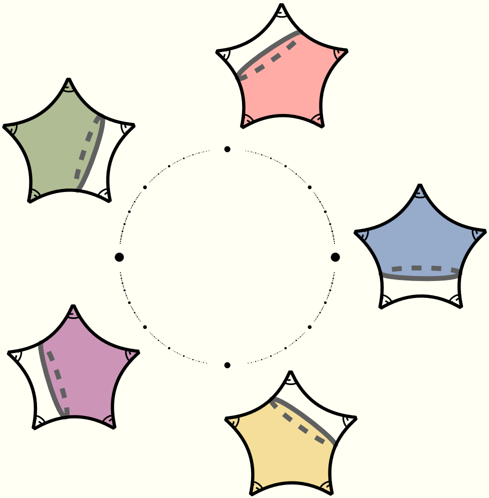
z4-011

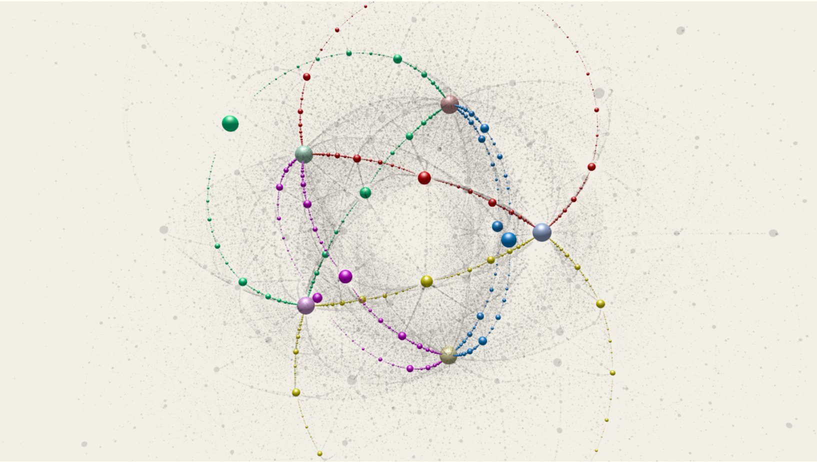
Rings

Rings



Rings





pmls05-071

Contact

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