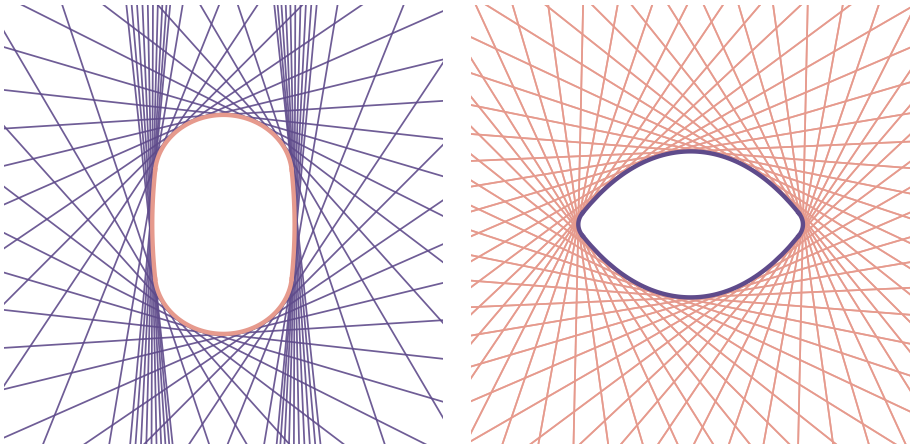
Math 569 – Flag Varieties: Geometry, Topology, and Dynamics – Fall 2024

Emily Dumas



1. The basics

Course sites	Main: dumas.io/teaching/2024/fall/math569/ Blackboard: uic.blackboard.com/ultra/courses/_272982_1/outline
Lectures	MWF 12:00-12:50pm in Taft Hall 316
Instructor	Emily Dumas (she/her) <edumas@uic.edu>
Office hours	Mon 1pm and Fri 10am
Office location	3108A BSB
Zoom office	https://uic.zoom.us/my/edumas
Instructor home page	https://dumas.io/

2. Course content

This is a graduate topics course on the generalized flag varieties associated to semisimple complex or real Lie groups. While these objects are often approached from the perspective of algebraic geometry, in this course we will mainly focus on topological, differential-geometric, and dynamical aspects.

The course will begin with a discussion of the classical partial and full flag varieties as generalizations of projective spaces and Grassmannian manifolds. We will then introduce the generalized flag varieties associated to semisimple Lie groups. We will study smooth and complex structures on these compact manifolds, describe their topology in terms of a natural CW structure (the Schubert cells), and compute topological invariants such as the homology groups and cohomology ring.

Next we will place flag varieties into a larger context by relating them to boundaries (or parts of boundaries) of symmetric spaces of noncompact type. Here a central theme will be the interplay between the G-invariant geometries of these two spaces: The Riemannian geometry of the nonpositively curved, complete symmetric space and the non-Riemannian, higher-order geometric structure of the flag variety.

Finally we will turn to studying dynamics on flag varieties, first for individual automorphisms and then for discrete groups of automorphisms. We will emphasize the concept of proximality and the way it gives rise to a well-behaved notion of "limit set". We will introduce the notion of Anosov subgroups, which are an important and dynamically well-behaved class of discrete subgroups of Lie groups introduced by François Labourie. Finally we will discuss a characterization of the Anosov property in terms of dynamics on the flag variety that was established by Kapovich, Leeb, and Porti.

3. Prerequisites

The course is intended for graduate students who have some knowledge of differentiable manifolds, algebraic topology, and Lie groups. In most cases UIC's year-long geometry and topology prelim sequence (Math 547 & Math 549) contains the necessary material.

Graduate students who are interested in the topic of the course but have a few gaps in their knowledge of the prerequisites may still find the course useful and are certainly welcome. As detailed in Section 9, everyone who is registered will be asked to make a short presentation at the end of the semester. I will be happy to work with each student to help find a topic suited to their background and interests.

4. Books and other resources

There are no required textbooks or other materials. The course web site contains a list of books and other resources where students can read more about the topics discussed in lecture. Such reading is not a required element of the course.

5. How the course is delivered

This a **synchronous in-person class held on campus**. This means that unless a change in modality is required by a university policy change or an unforeseen circumstance affecting my availability, the course will be held at the scheduled time in Taft Hall 316. Section 11 describes some contingency plans in case of my unexpected absence.

I will present most material by writing on the board. My lecture notes will be posted on the course web site. These notes are not intended to be as polished or complete as a textbook, and for additional detail students are encouraged to consult the optional texts.

6. Remote participation and lecture video recording

A **zoom meeting** will be held during lecture to provide a participation option for students who cannot come to campus for any reason. While I will take all reasonable steps to make material presented in class accessible in the zoom meeting (e.g. through camera position, size of writing on the board, and use of a microphone), providing both remote and in person participation options always requires some trade-offs. In such cases the in-person experience will be prioritized.

Please don't regularly attend via zoom if coming to campus is an option for you. Teaching to a nearly empty classroom is very difficult.

The lecture zoom meetings will be recorded (audio and video) and these **lecture recordings will be posted** to the Blackboard site. Each lecture video will be available to students enrolled in the course for 180 days from the date of recording. In most cases the recordings will only include the instructor, but students may appear incidentally from time to time (for example when asking questions).

7. The course web sites

The main course web site is public and will remain available to everyone after the course ends:

https://www.dumas.io/teaching/2024/fall/math569/

A few materials will only be available to UIC students and/or during the fall 2024 semester. These include lecture zoom meeting links, lecture video recordings, and the schedule of end-of-semester presentations. Such materials will be posted to the course site on Blackboard:

https://uic.blackboard.com/ultra/courses/_272982_1/outline

Each site has a link to the other.

8. Important dates and deadlines

Entries in **bold** are specific to this course. The others are are taken directly from the UIC academic calendar and are shown to put course-specific dates in context.

Aug 26	Mon	First lecture
Sep 2	Mon	No class (Labor day)
Nov 18	**Mon**	**Presentation topics due**
Nov 27–29	Wed–Fri	No class (Thanksgiving)
Dec 6	Fri	Last lecture
Dec 9–13	Mon–Fri	Final exam week
?	**?**	**Student presentations held in scheduled final exam period**
Dec 17	Tue	Departmental deadline for instructor to submit grades
Dec 23	Mon	Course grades become available on my.uic.edu

9. Presentations

Each student enrolled in the course will be required to give a 20-minute in-class presentation at the end of the semester on a topic related to the course material. Topics of such presentations must be selected by **Monday, November 18**.

These presentations will be held during the final exam period allocated to this course by the UIC registrar. (I will post an announcement as soon as the final exam schedule is released.) If the number of students enrolled necessitates additional presentation time slots, one or more of the lectures in the last week of the semester will be used as well. The presentation schedule will be posted in advance on the course Blackboard site.

Each student will receive an evaluation of their presentation by email. It will consist of:

- A paragraph of feedback on the content and presentation,
- A numerical score (out of 100 points), and
- A letter grade

10. Attendance policy

Students will of course get the most out of the course by attending all of the lectures. However, following public health guidance, students who are sick or experiencing symptoms of illness should never participate in in-person activities.

Attendance in lectures will not be graded or recorded.

11. Absence contingency plans

11.1. **Student absent from lecture.** Any student who misses a lecture should review the posted lecture notes and/or the lecture video recording.

11.2. **Instructor absent from lecture.** If I cannot lead a course lecture in person, one of these things will happen:

(1) Online synchronous lecture: Class will be held entirely on zoom, using the same meeting link provided for remote participation in lectures.

(2) Cancellation: The course meeting will be canceled, with the next lecture containing the content planned for the canceled one.

(3) Asynchronous lecture delivery: A lecture video will be recorded and provided to all students.

(4) In-person substitute: Another instructor will lead the meeting as usual.

These options are roughly ordered from most likely to least likely. In any case, an announcement of the specific plan will be made with as much advance notice as possible. It is expected that instructor absences will be rare.

12. Policy on missed or late work

There are very few required, scheduled course activities—only the deadline to select a presentation topic and the 20-minute presentation itself have these properties. Both allow a lot of flexibility to prepare in advance. Thus, if you know in advance that you have a conflict with one of these, you should not take Math 569.

If you miss one of these scheduled activities due to an unforeseen situation, or if something arises during the semester that is likely to cause you to miss one of them, contact me as early as you can to discuss the situation and work out an equitable solution.

13. Academic honesty

Students will not be submitting written work in this course, so the university policies on academic honesty are not likely to be relevant. However, everyone should be aware of the UIC Student Disciplinary Policy.

14. Course grade

Grades will be assigned based on the end-of-semester presentations, the evaluation of which is described in Section 9.

15. Communication with course staff

Instructor office hours will be held in person as long as university policy allows this. On request, I can also meet with students on Zoom during office hours. Students who cannot attend office hours but want to meet with me can request an appointment.

Outside of course meetings and office hours, **email** is the best way to contact me.

16. Learning environment

16.1. **Inclusive community.** UIC values diversity and inclusion, as do I. You can read more about this at

`https://diversity.uic.edu/`

Regardless of age, disability, ethnicity, race, gender, gender identity, sexual orientation, socioeconomic status, geographic background, religion, political ideology, language, or culture, we expect all members of this class to contribute to a respectful, welcoming, and inclusive environment for everyone. If aspects of this course result in barriers to your inclusion, engagement, accurate assessment, or achievement, please notify me as soon as possible.

16.2. **Sharing preferred name and pronouns.** By editing your user profile in Blackboard, you can specify a preferred name and your personal pronouns. These are always visible to your course instructors, and you can choose whether to make your Blackboard profile visible to other students. Alternatively, you are welcome to share this information in a lecture meeting or to contact me directly (by email or in a meeting).

16.3. **Conduct policy.** Group meeting environments (like Math 569) work best when norms of behavior are explicitly spelled out. Everyone in the course is expected to:

(1) Be present by refraining from use of electronic devices for purposes unrelated to course work.
(2) Be respectful of the learning space by avoiding side conversations and unnecessary disruptions.
(3) Use preferred names and personal pronouns of course students and staff (if and when they are shared with you).
(4) Assume good will in all interactions, even in disagreement.
(5) Be open to change and receptive to both positive and critical feedback from peers and course staff.
(6) Be mindful of one another's privacy, for example by refraining from audio or video recording unless explicit permission is given by all participants.

17. University Wellness Resources

Please be aware of the following programs at UIC. All of the programs listed below are available to graduate students.

- **Counseling Services** – Free and confidential. https://counseling.uic.edu/
- **U & I Care Program** – University program providing assistance for students dealing with with personal hardships. https://dos.uic.edu/student-assistance/uicare/
- **Campus Advocacy Network** – Under Title IX, you have the right to an education free from any form of gender-based violence or discrimination. Reports can be submitted to TitleIX@uic.edu. For more information or confidential victim services and advocacy, see http://can.uic.edu/.

18. University Policies

UIC requires that every syllabus mention the following university policies.

18.1. **Academic deadlines.** The UIC academic calendar can be found at:
http://catalog.uic.edu/ucat/academic-calendar/
In particular this calendar includes the deadlines for adding and dropping courses. Note that this semester (fall 2024) includes the first implementation of a recent university policy change: **Lecture will not be held on the Wednesday before Thanksgiving (Wed Nov 27).**

18.2. **Standards of conduct.** All UIC students are required to abide by the rules and standards of conduct described in the Student Disciplinary Policy (https://go.uic.edu/DisciplinaryPolicy).

18.3. **Disability accommodation.** The University of Illinois at Chicago UIC is committed to full inclusion and participation of people with disabilities in all aspects of university life. Students who face or anticipate disability-related barriers while at UIC should connect with the Disability Resource Center (DRC) by visiting drc.uic.edu, by emailing drc@uic.edu, or by calling (312) 413-2183 to create a plan for reasonable accommodations. In order to receive accommodations, students must disclose disability to the DRC, complete an interactive registration process with the DRC, and provide their course instructor with a Letter of Accommodation (LOA). Course instructors in receipt of an LOA will work with the student and the DRC to implement approved accommodations.

18.4. **Religious holidays.** The UIC Senate Policy on religious holidays is as follows:

"The University of Illinois Chicago faculty shall make every effort to avoid scheduling examinations or requiring student projects be turned in or completed on religious holidays. Students who wish to observe their religious holidays shall notify the faculty member by the tenth day of the semester of the date when they will be absent unless the religious holiday is observed on or before the tenth day of the semester. In such cases, the students shall notify the faculty member at least five days before the date when they will be absent. In cases when the exact date(s) of the religious holiday is/are not known at the start of the semester, the student

should notify the faculty member as soon as the exact date is known. Students should be asked to report if such situations might occur within the first four weeks of the semester. The faculty member shall make every reasonable effort to honor the request, not penalize the student for missing the class, and if an examination or project is due during the absence, give the student an exam or assignment equivalent to the one completed by those students in attendance. If the student feels aggrieved, they may request remedy through the campus grievance procedure."

The University Holidays and Religious Observances calendar can be found at:
http://oae.uic.edu/religious-calendar/

19. Revision history of this document

Any change to this document will be recorded here. Such changes are expected to be rare.

- 2024-08-23 Initial publication
- 2024-08-27 Add office hours
- 2024-12-02 Change pronouns
- 2024-12-02 Change office location