LECTURE 16

TREES

MCS 275 Spring 2023
David Dumas

RECENTLY

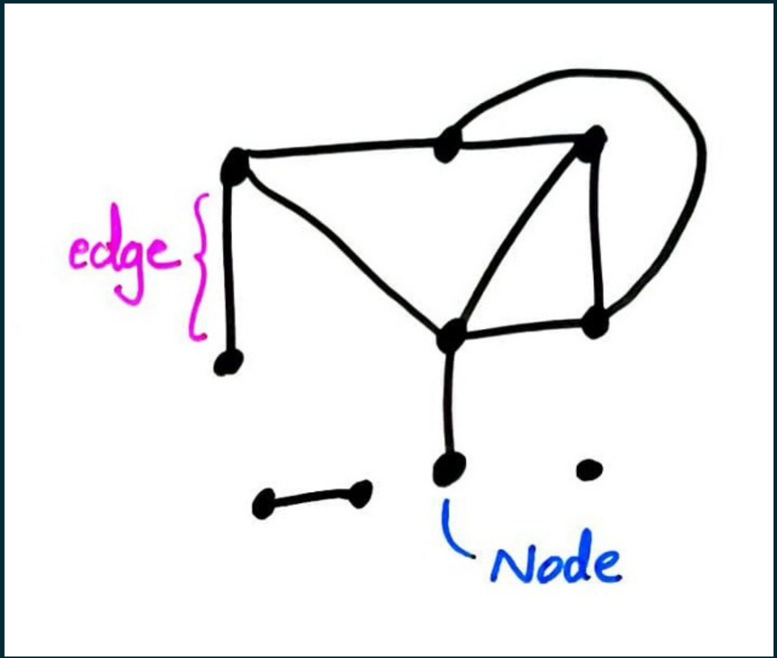
We've talked about some recursive algorithms

NEXT

Recursive **data structures**!

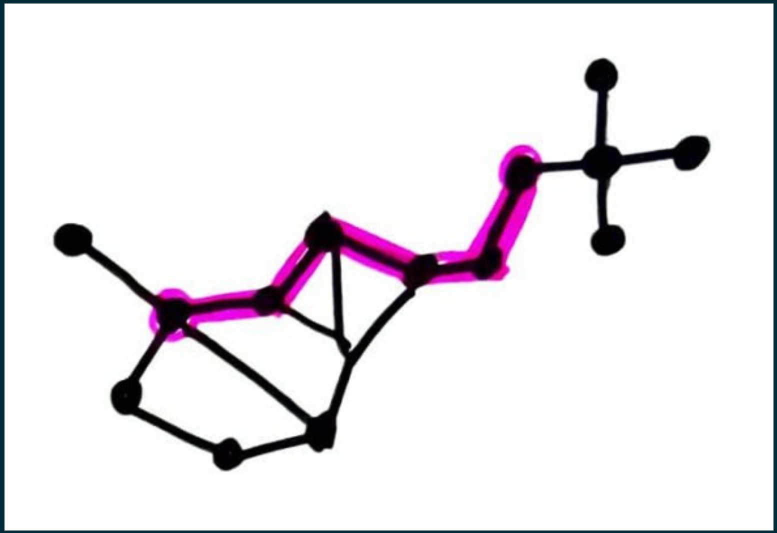
GRAPHS

In mathematics, a graph is a collection of **nodes** (or vertices) and **edges** (which join pairs of nodes).



CONNECTIVITY

A graph is **connected** if every pair of nodes can be joined by *at least* one path.



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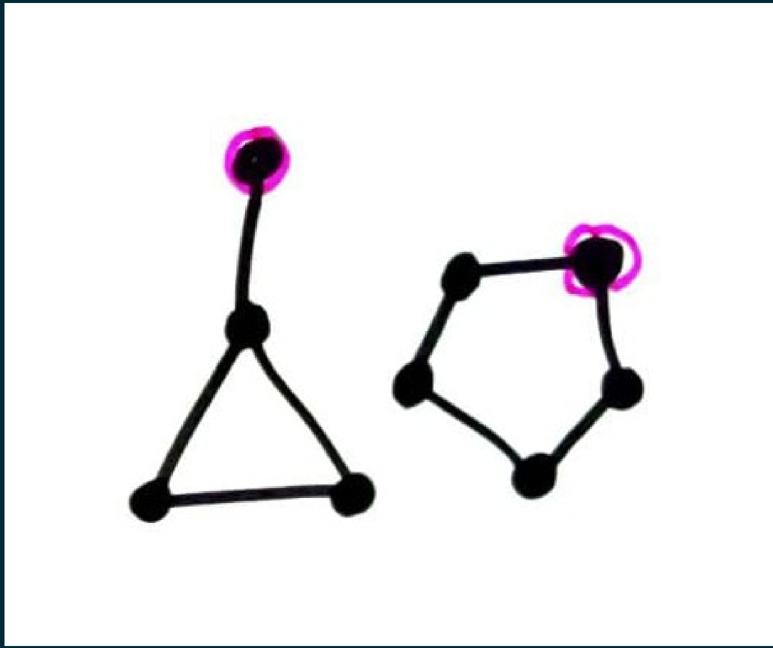

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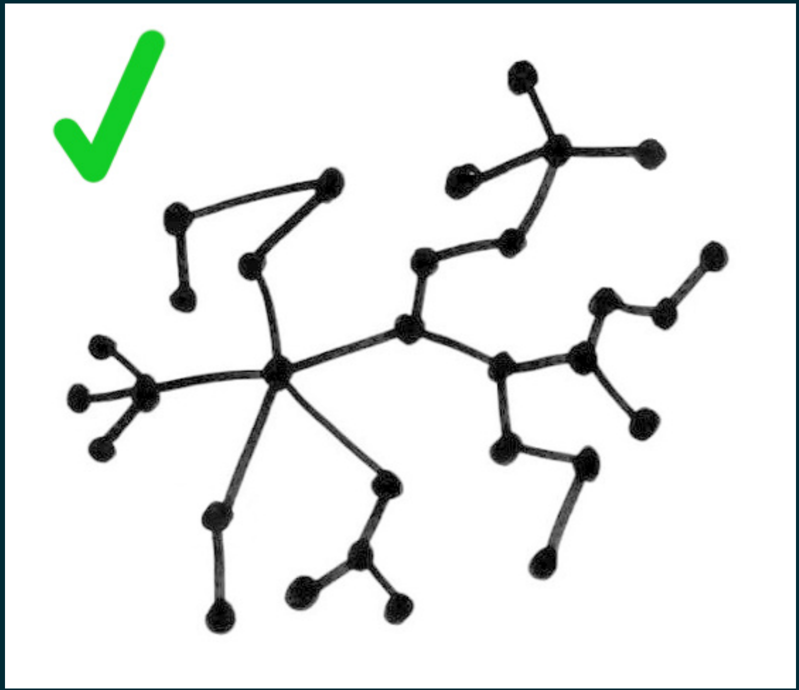

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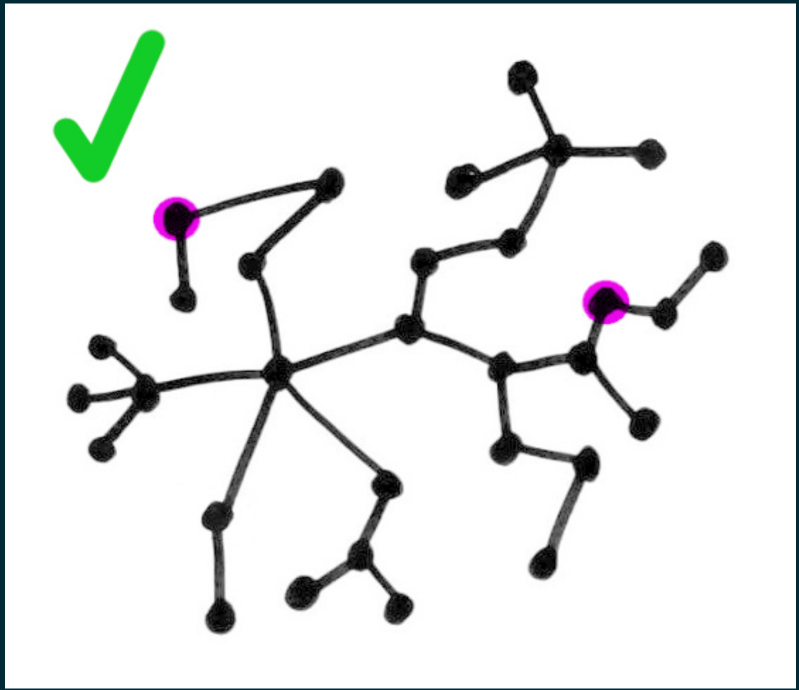

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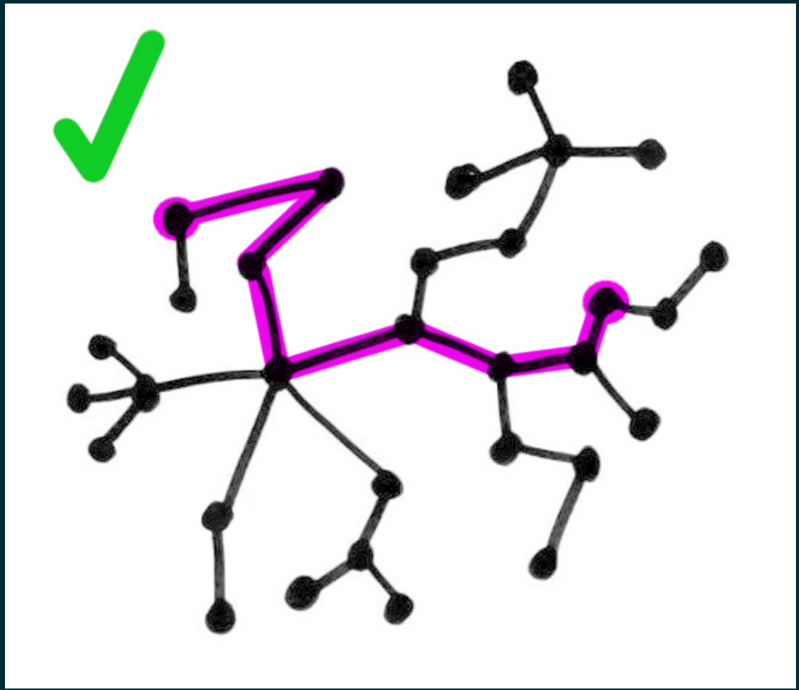

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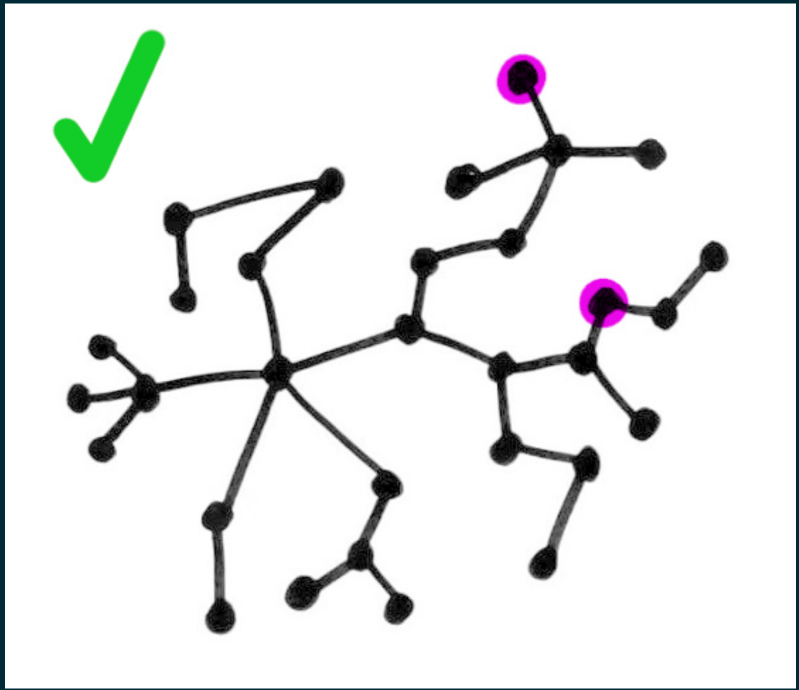

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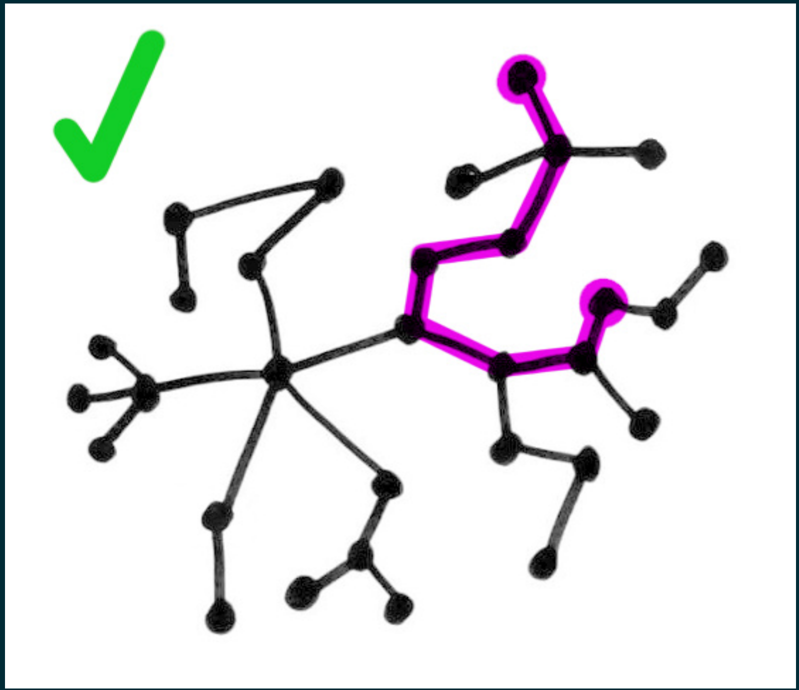

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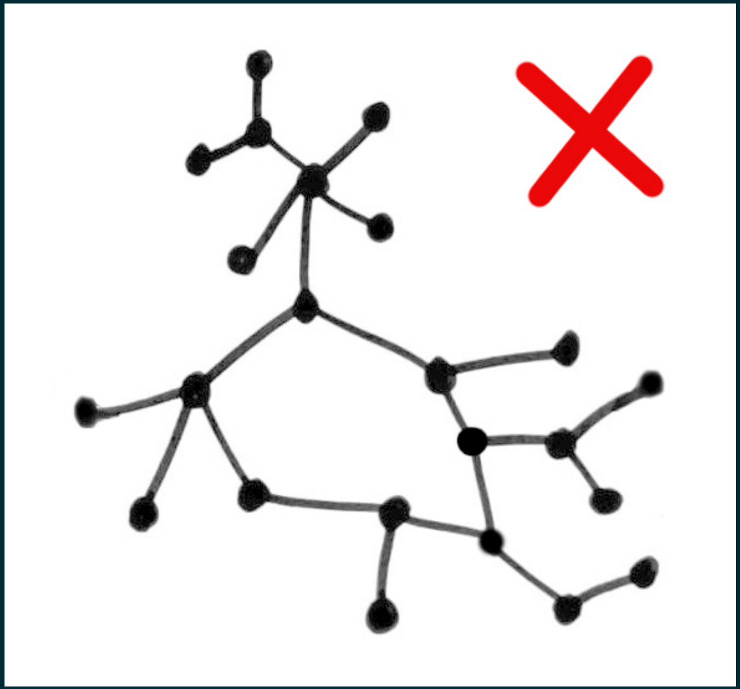

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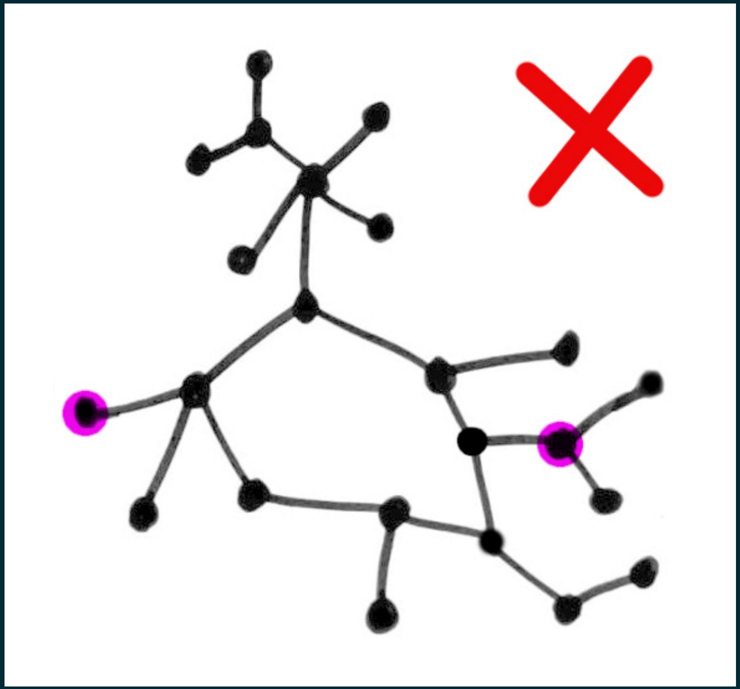

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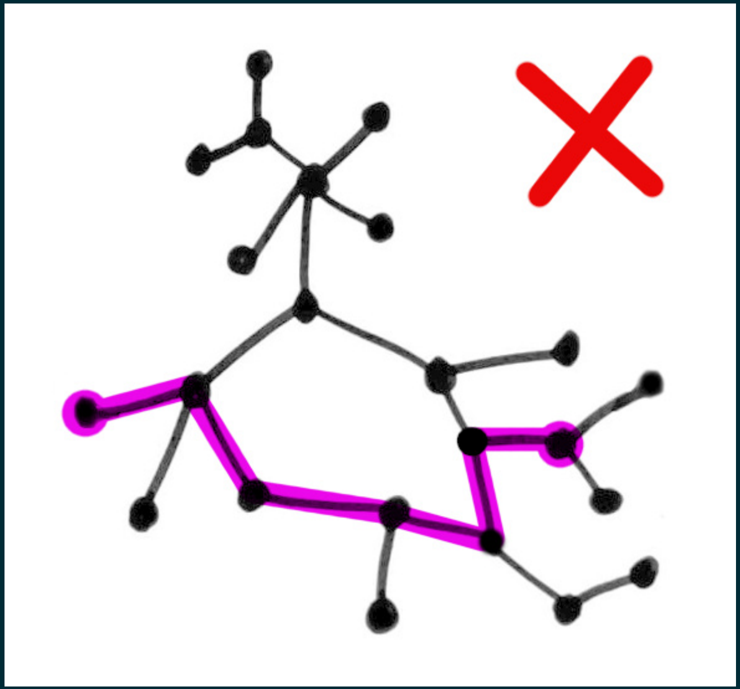

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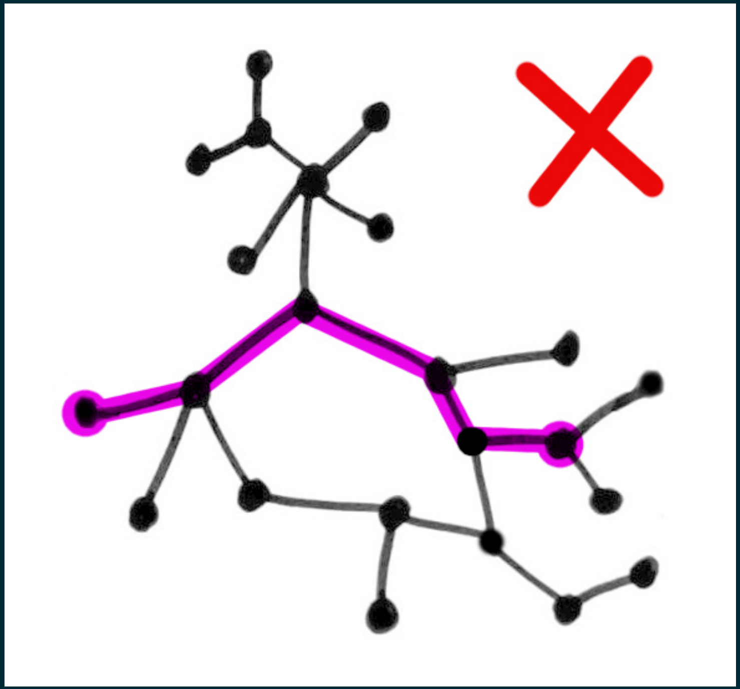
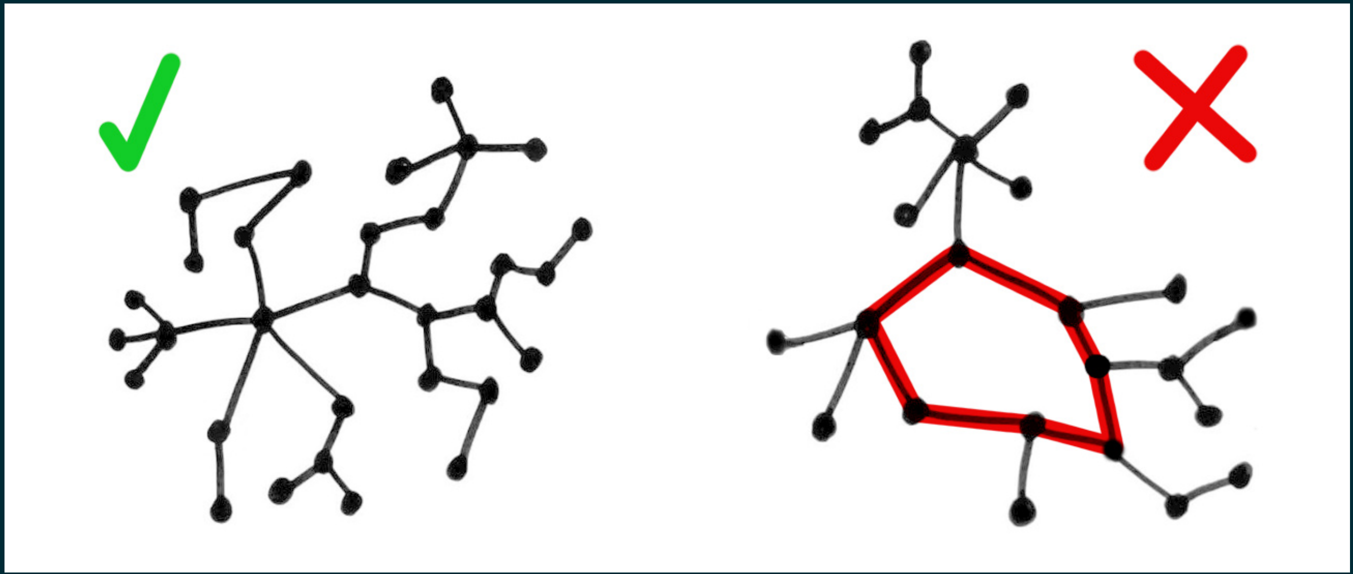


Equivalently, a tree is a **connected graph** with **no loops**.



Equivalently, a tree is a connected graph that becomes disconnected if any edge is removed.

(Exercise: Prove this is an equivalent definition!)

EXAMPLE

The random mazes produced by `maze.PrimRandomMaze(...)` can be seen as trees, with the nodes being the open squares and edges between nodes if the corresponding squares share an edge.

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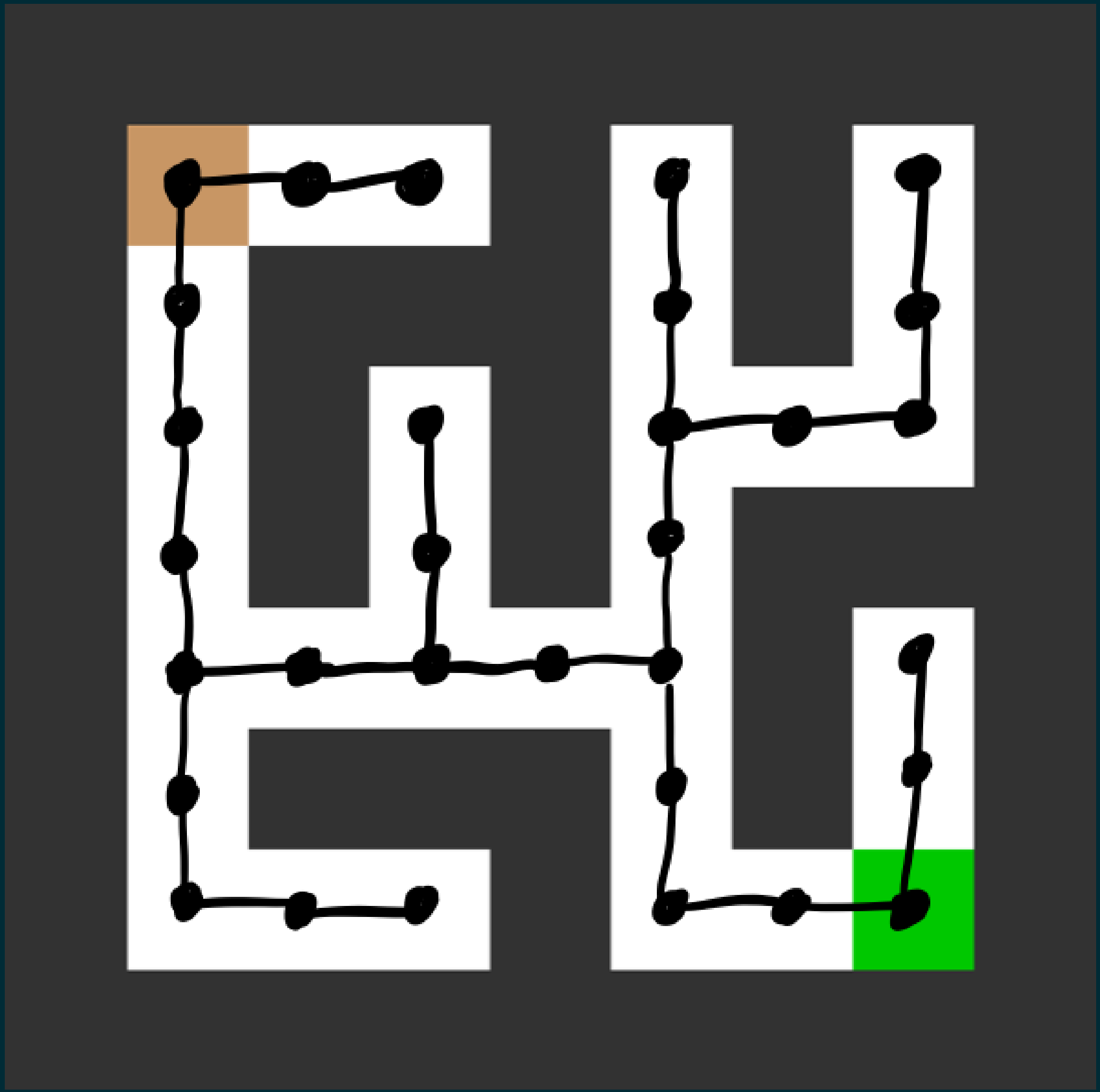

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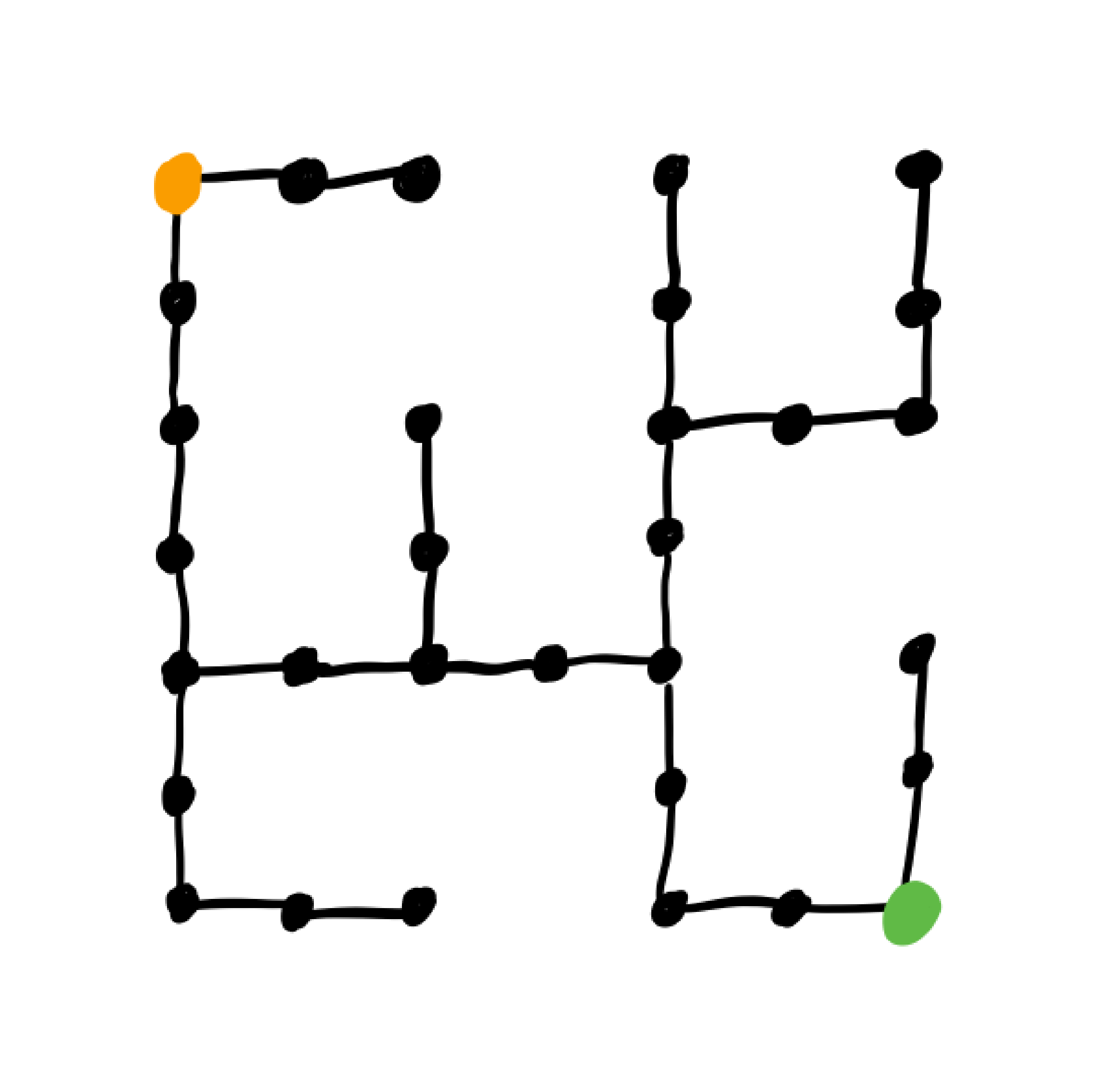

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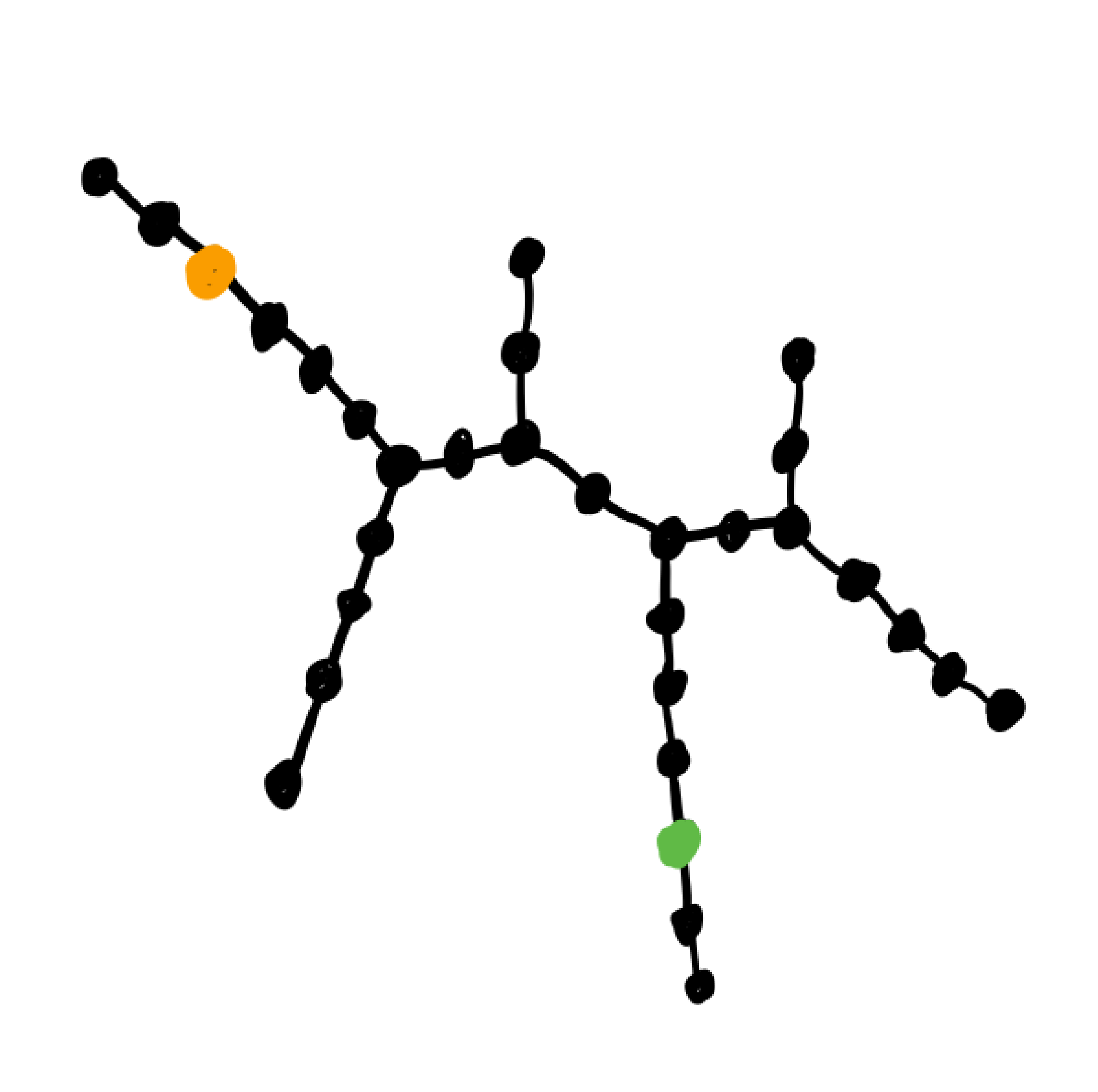
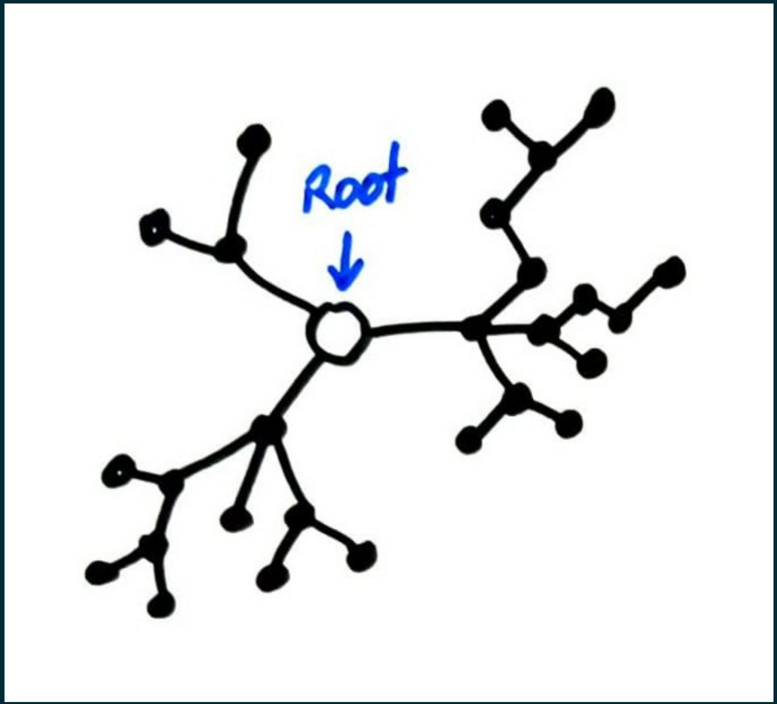


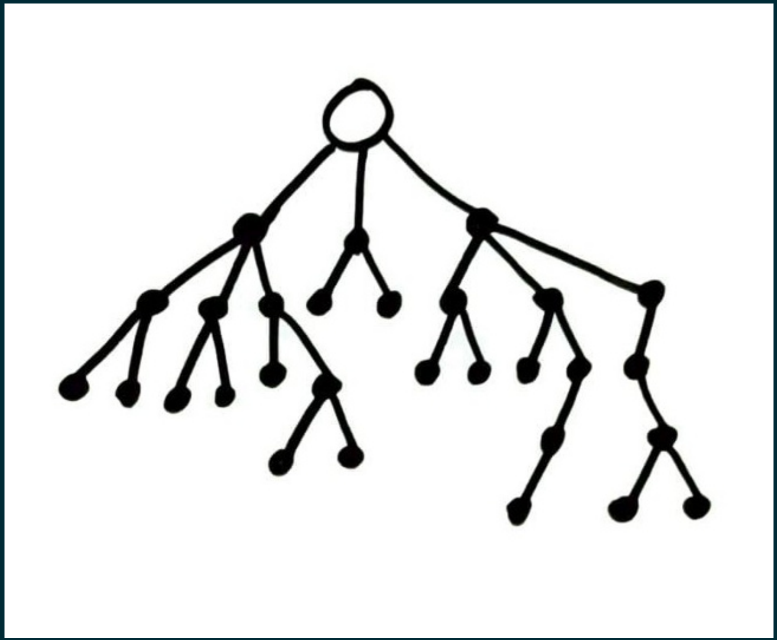
ROOTS AND DIRECTIONS

The trees considered in CS usually have one node distinguished, called the **root**.



There's nothing special about the root except that it is labeled as such. Any node of a tree could be chosen to be its root node.

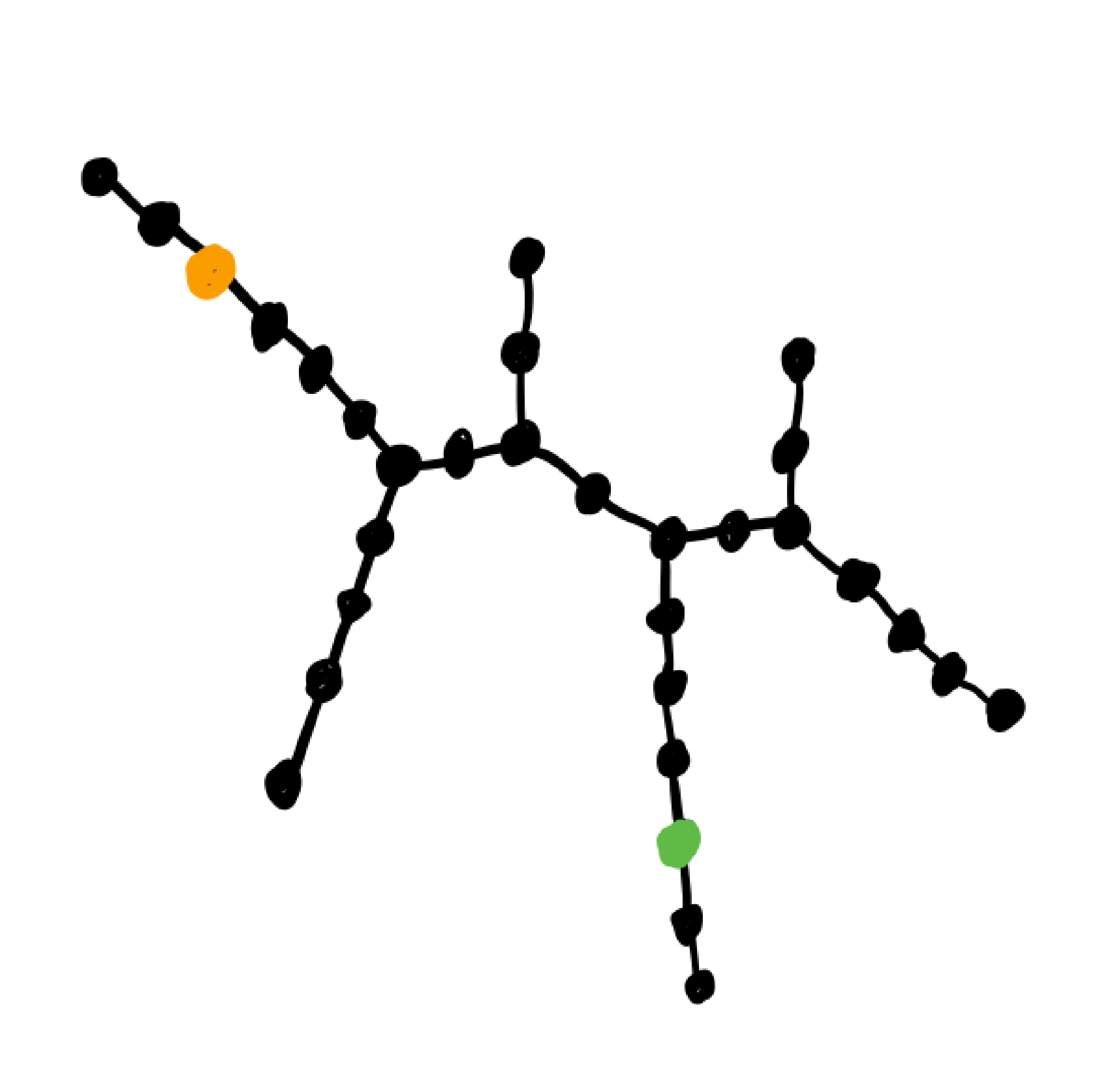
Such *rooted trees* are usually drawn with the root at top



and vertices farther from the root successively lower.

Such diagrams look like trees in the natural world.

THAT MAZE AGAIN



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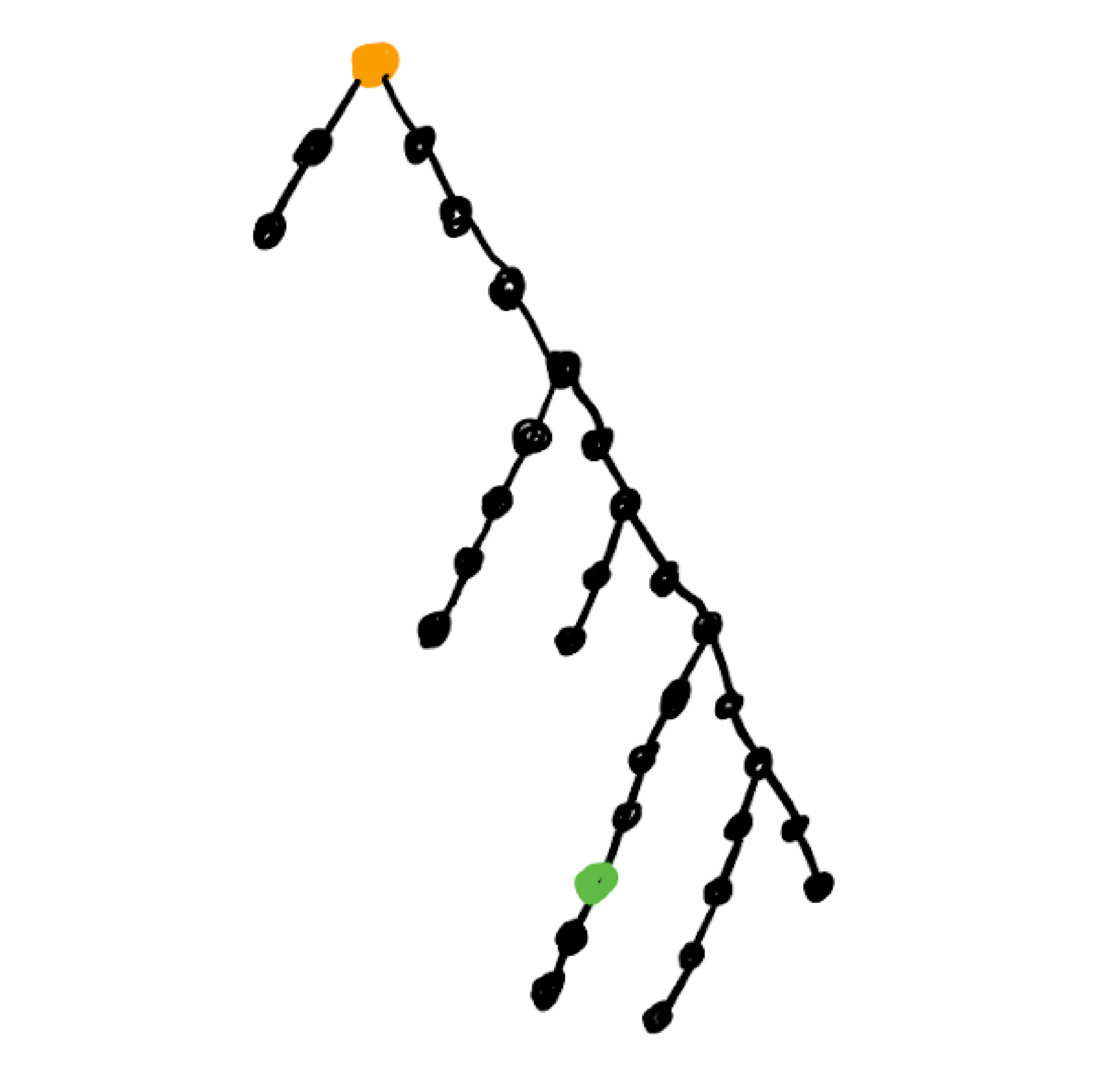
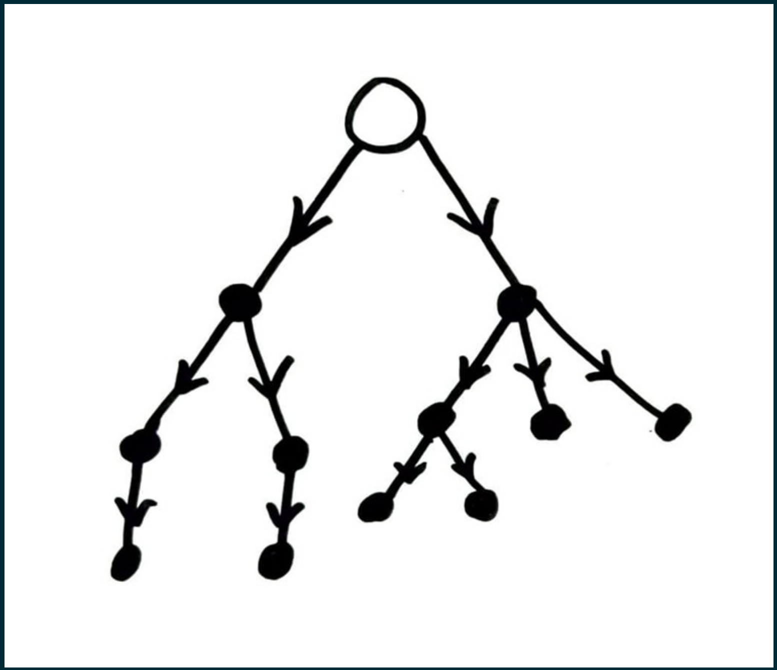
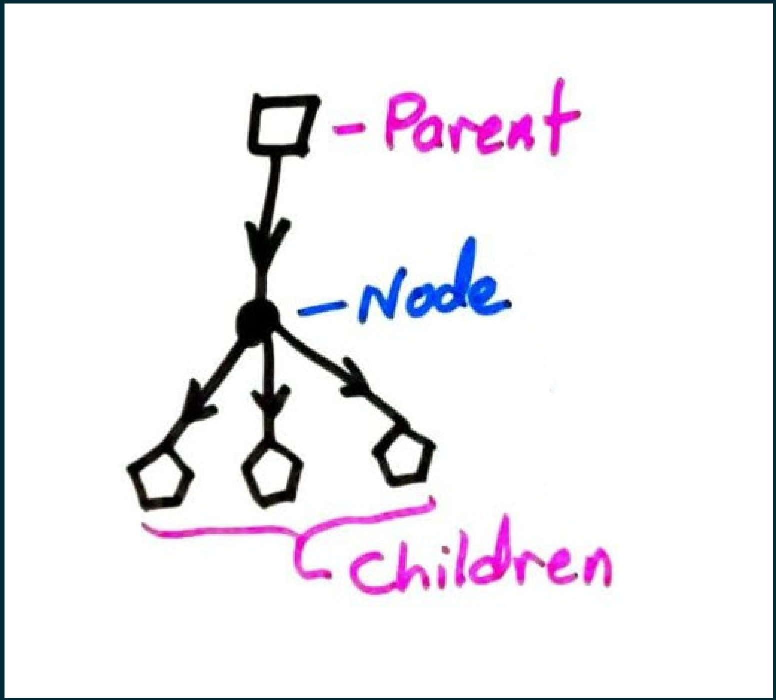


Choosing a root lets us orient all of the edges so they point away from it.



Hence the usual way of drawing a tree will have these arrows pointing downward.

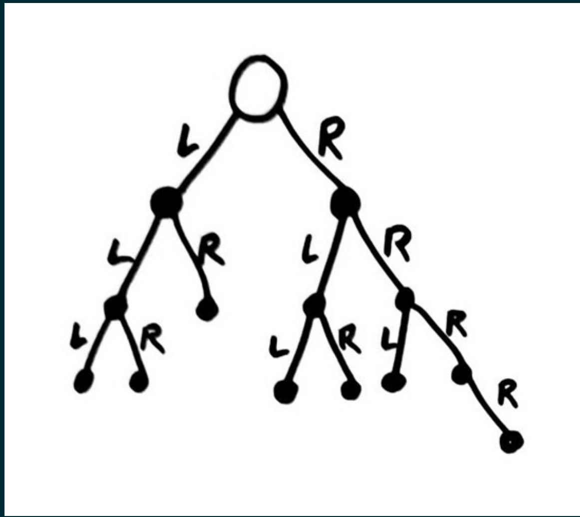
Each node (except the root) has an incoming edge, from its **parent** (closer to the root).



Each node may have one or more outgoing edges, to its **children** (farther from the root).

BINARY TREES

In CS, a binary tree is a (rooted) tree in which every node has ≤ 2 children, labeled "left" and "right".



Horizontal relative position is used to indicate this labeling, rather than explicitly writing it on the edges.

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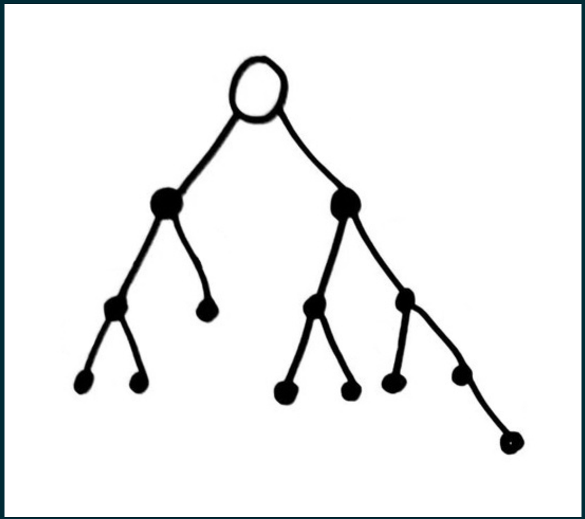

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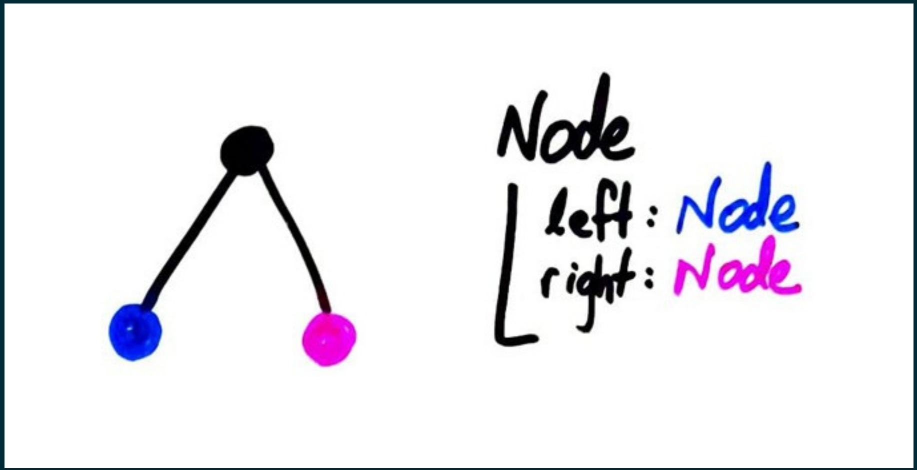
REPRESENTATION

How can we store a tree in Python?



Make a class `Node`, with attribues `left` and `right` that can be `None` or other `Node` objects.

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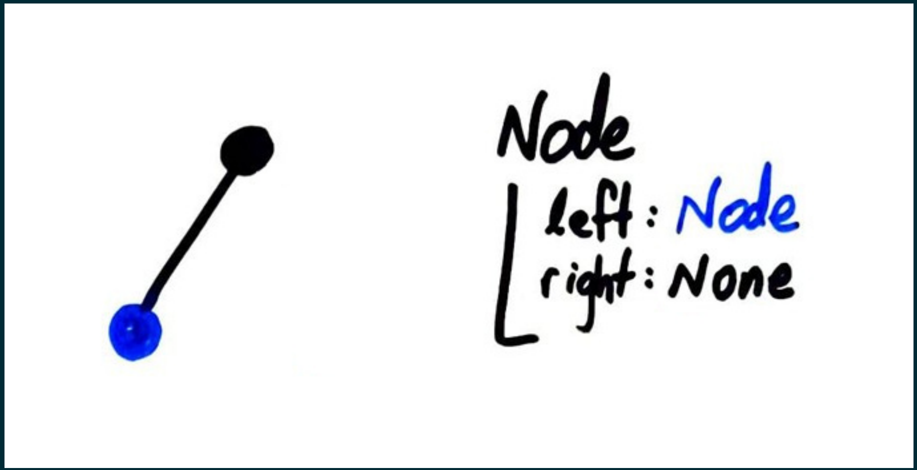

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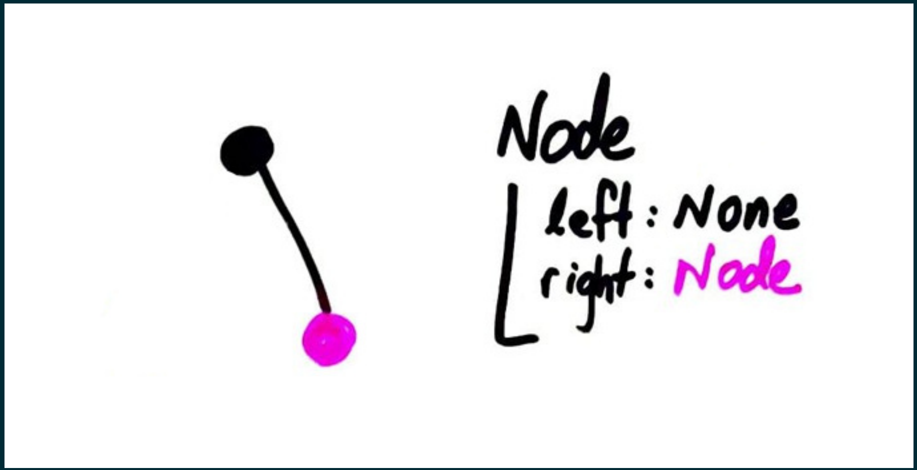

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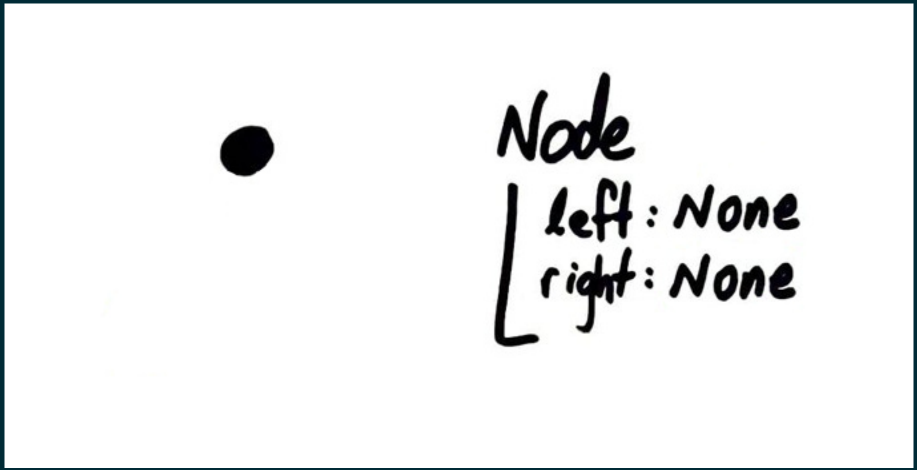

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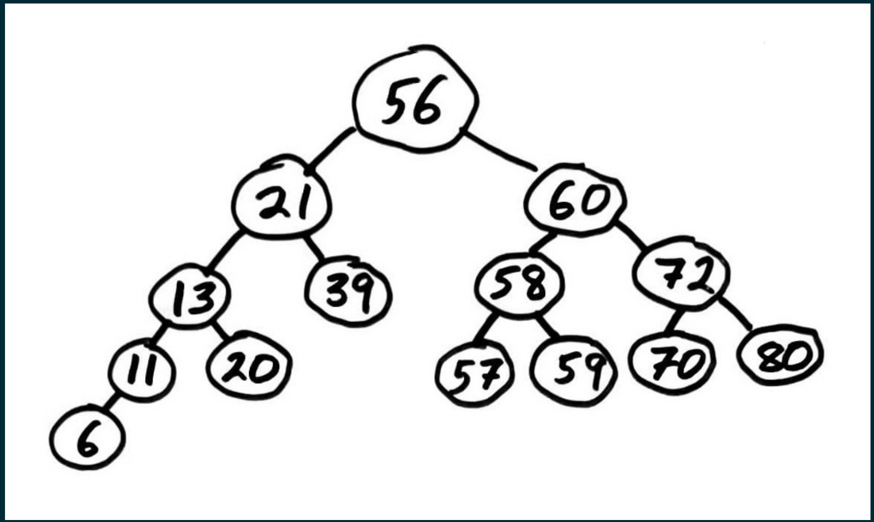
WHY?

We can also store additional information in the nodes of a binary tree. If present, this is called the **key** or *value* or *cargo* of a node.

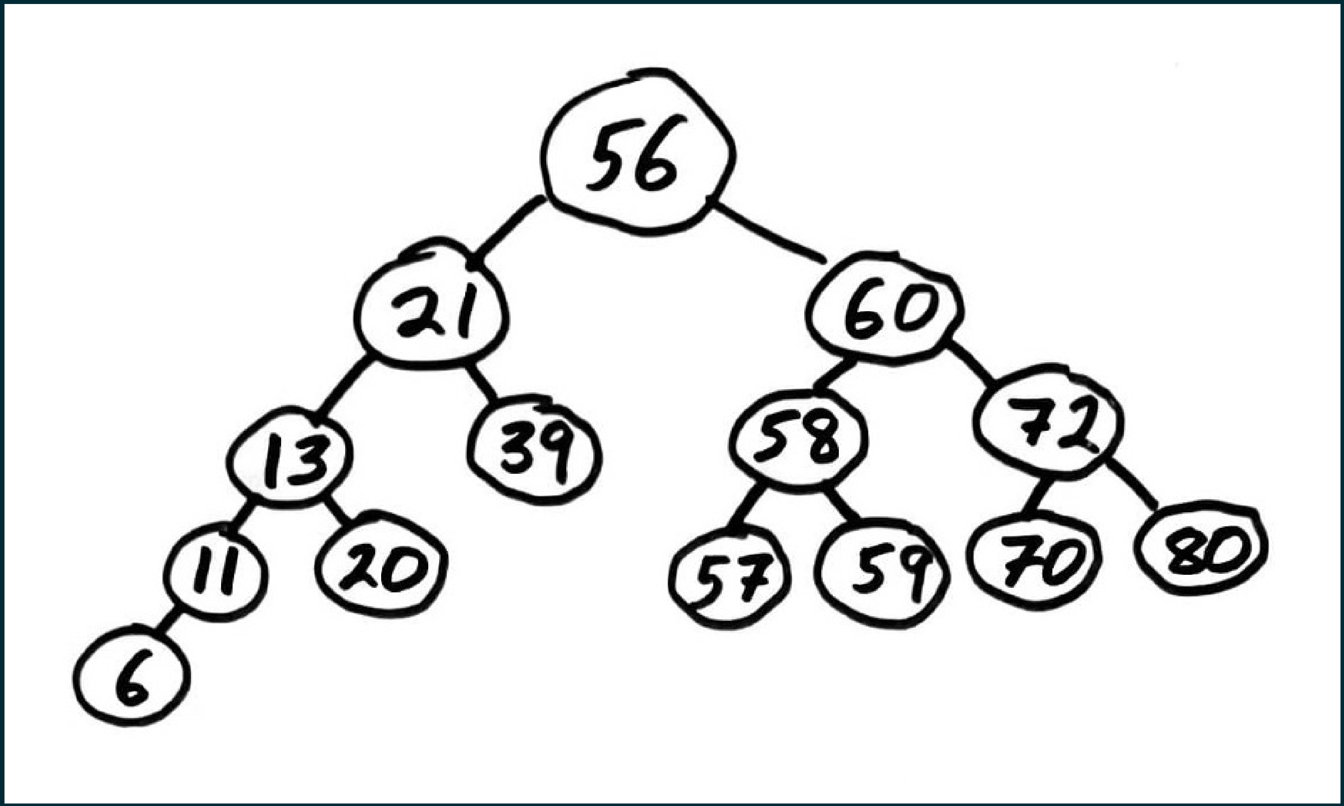
This turns out to be a very efficient data structure for many purposes. A lot of data can be accessed in a few steps from the root node.

BINARY SEARCH TREE

Stores numbers or other objects allowing comparison as node values. Enforce the rule: Anything in the subtree to the left is smaller or equal. Anything in the subtree to the right is greater.



This allows a natural way to check if a value is present with a game of "too high / too low".



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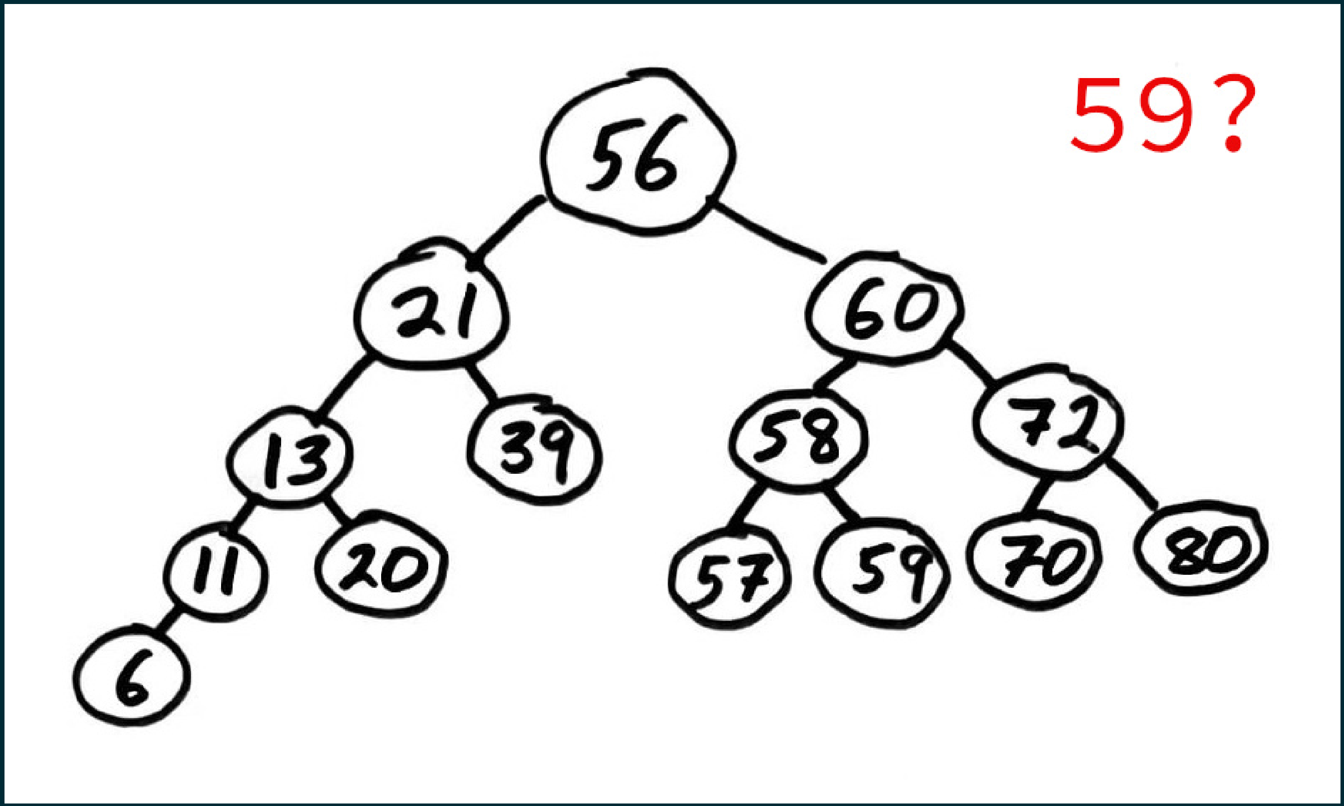

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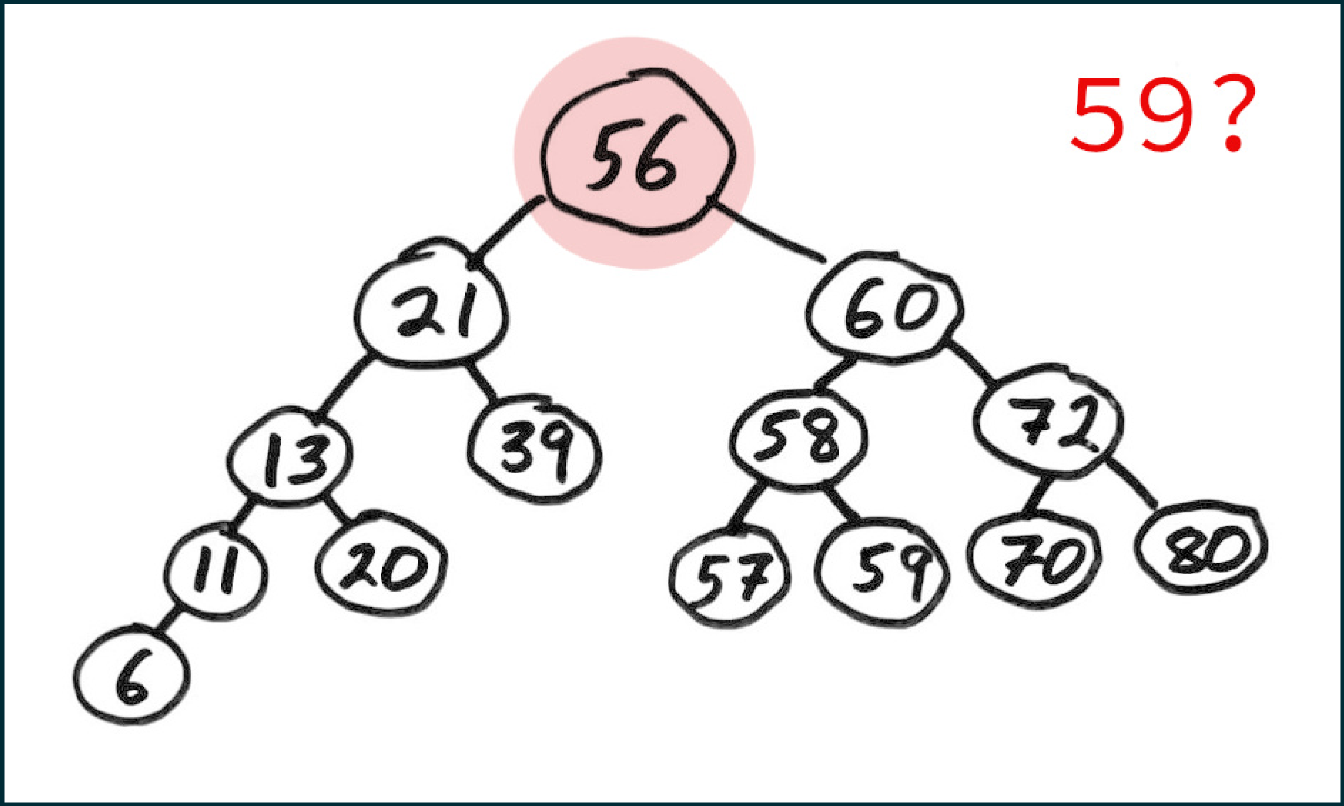

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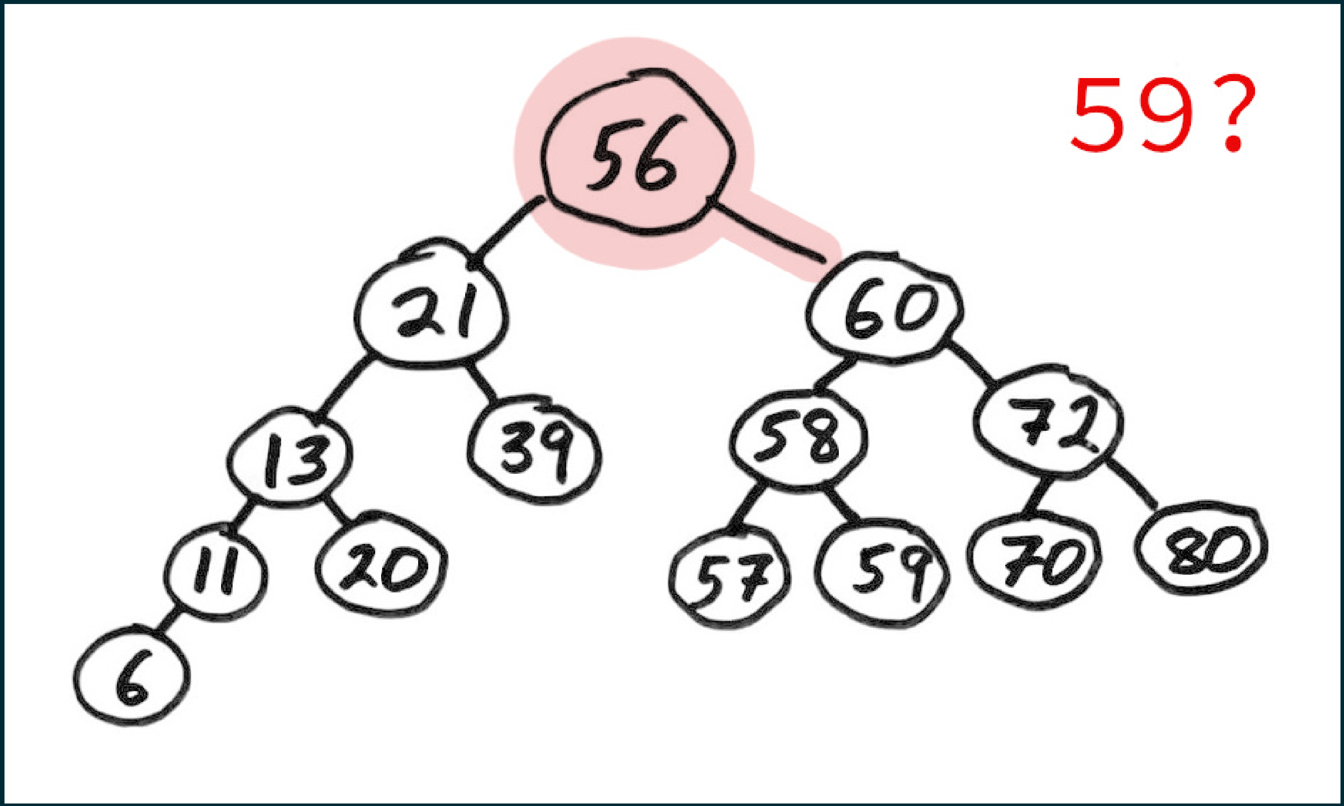

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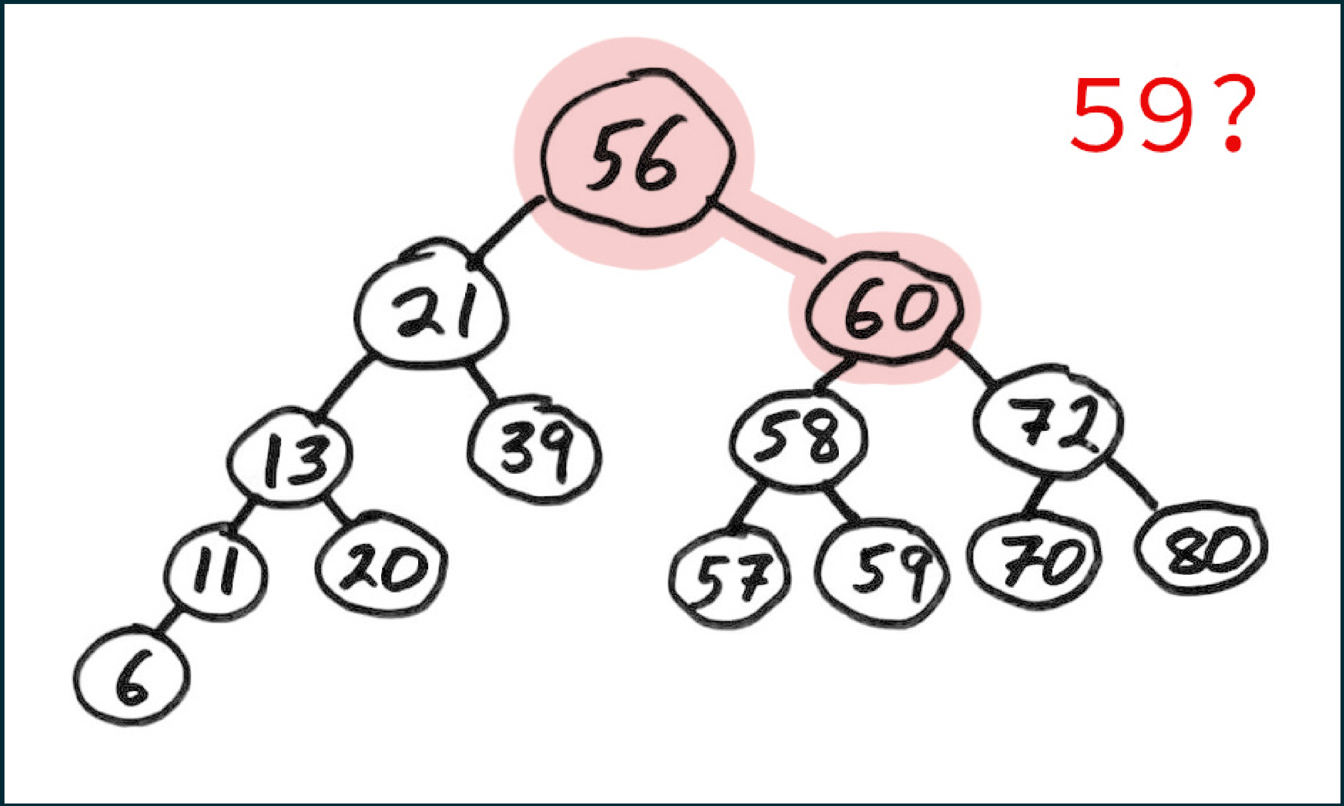

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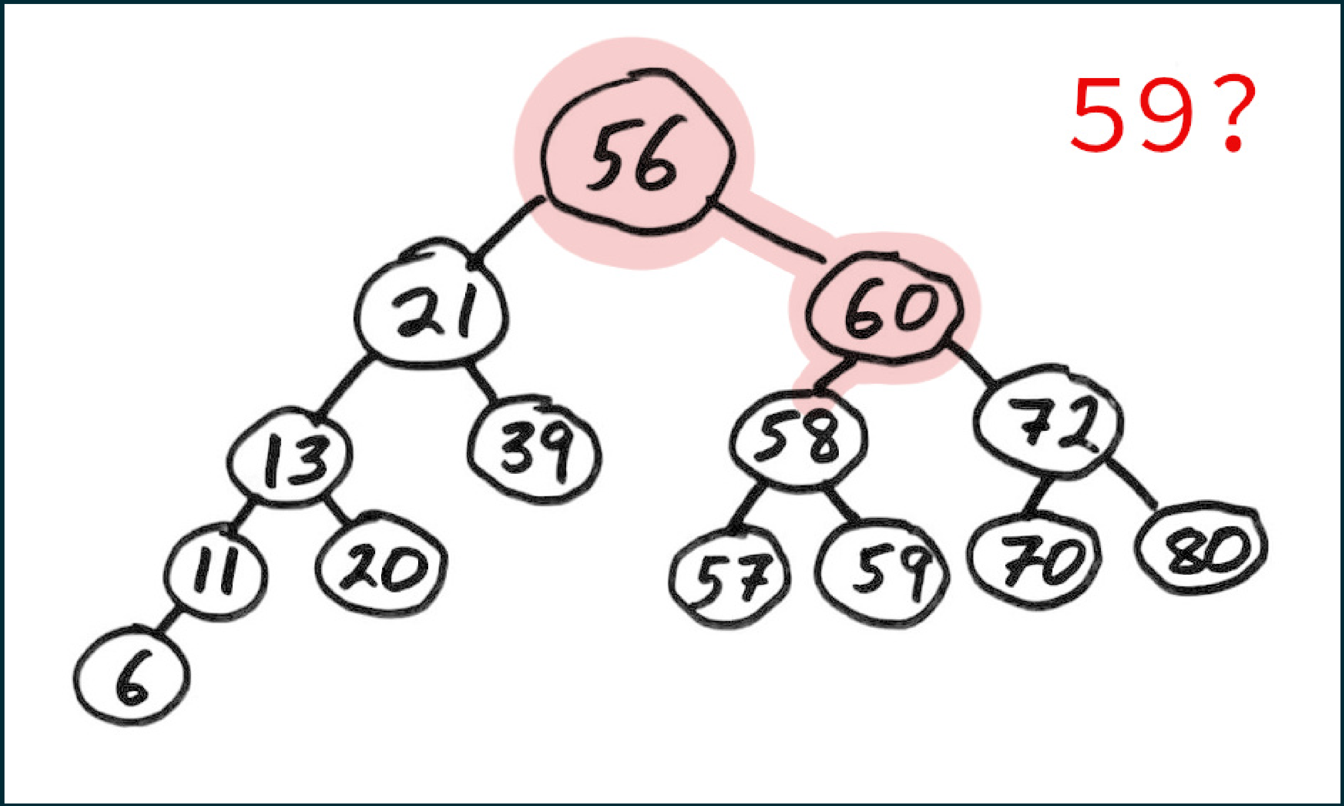

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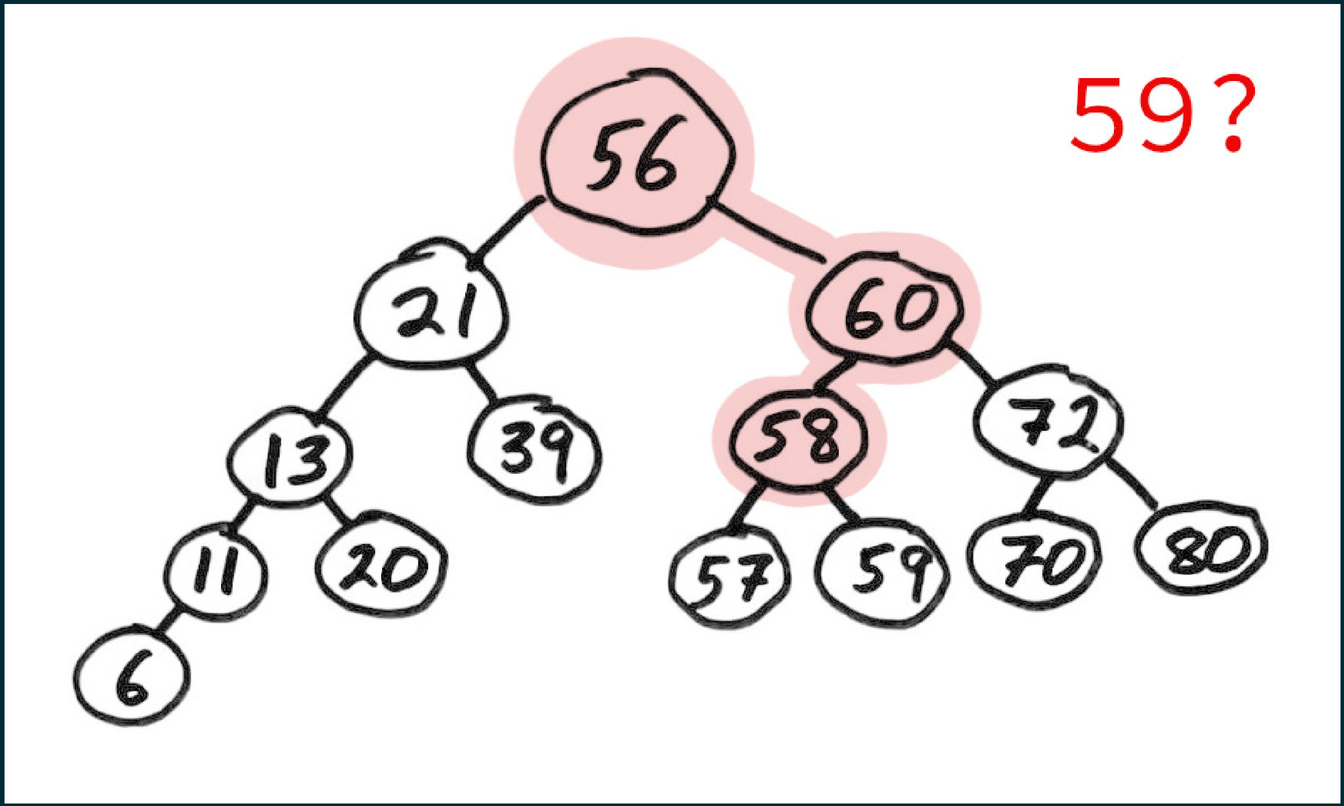

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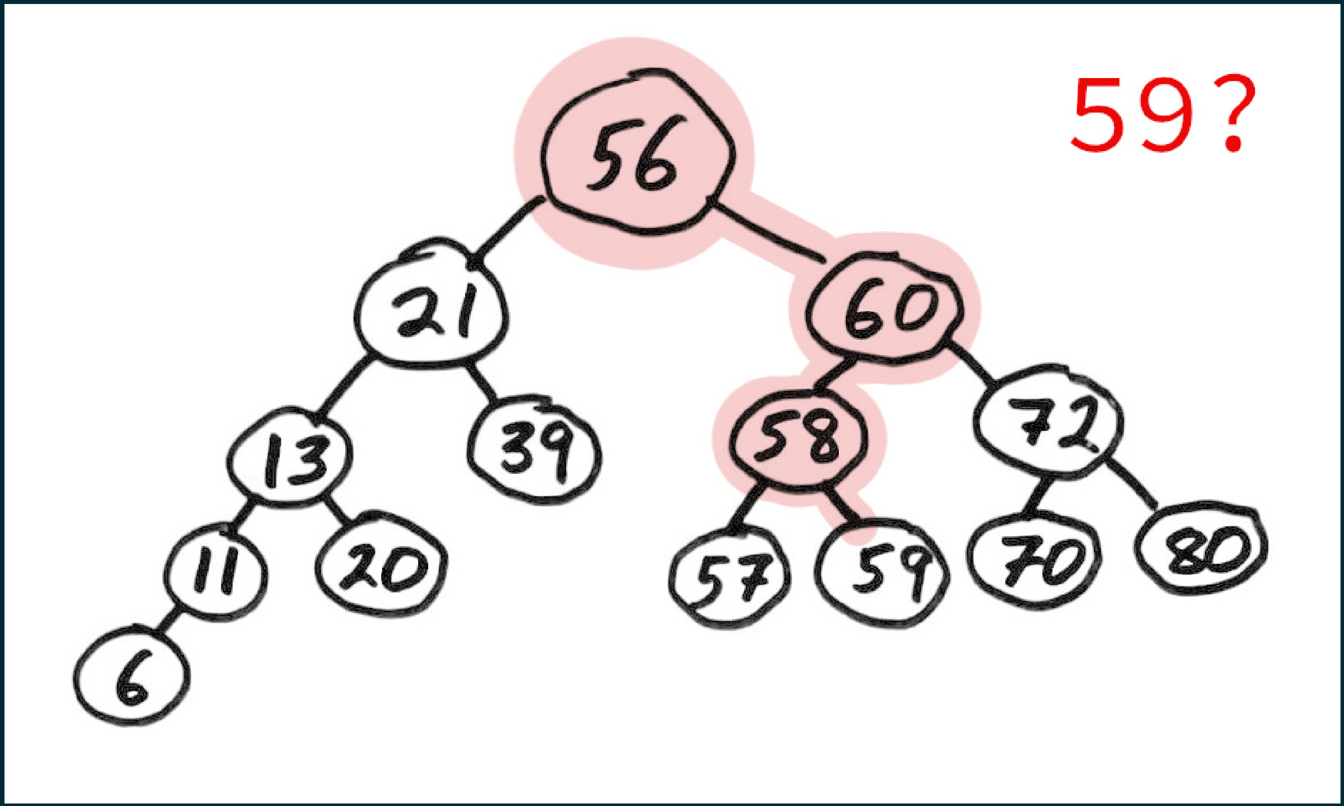

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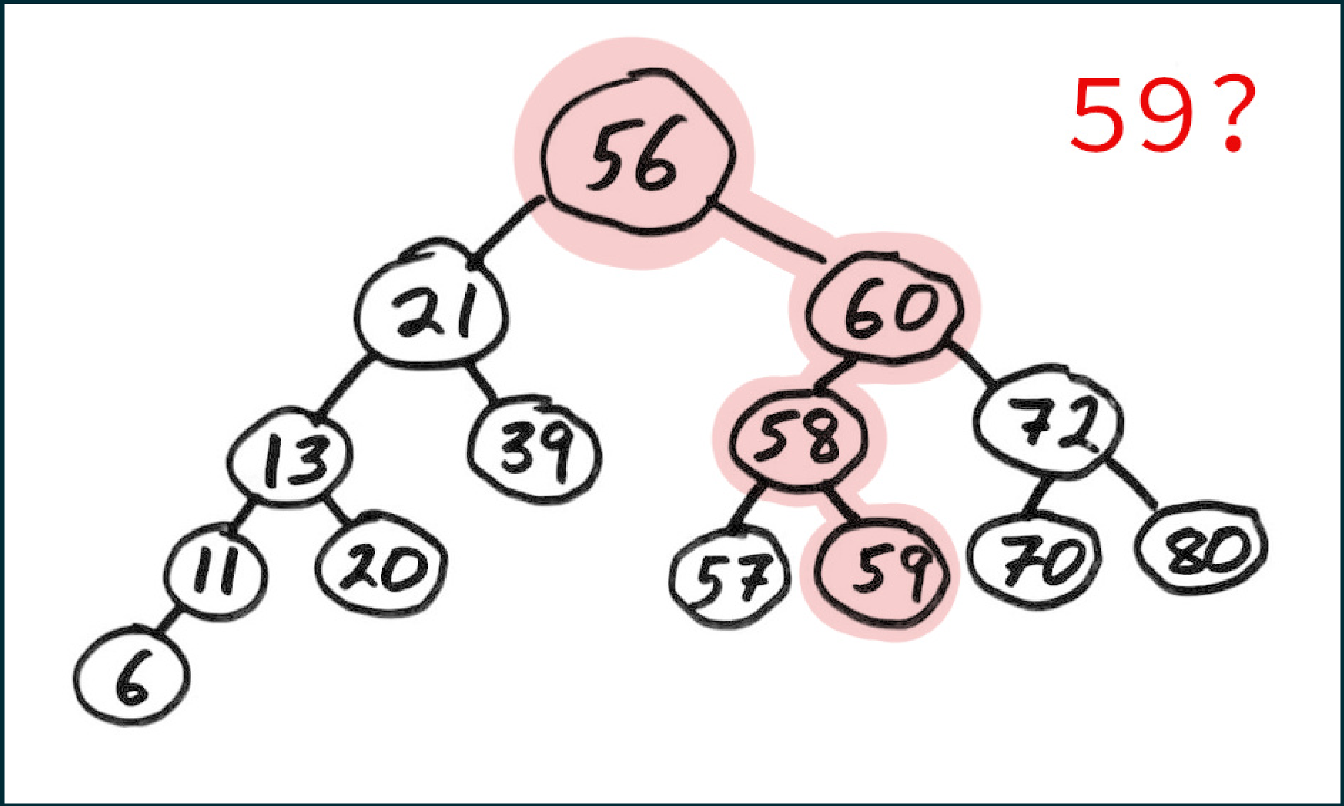

REFERENCES

- In optional course texts:
  - *Problem Solving with Algorithms and Data Structures using Python* by Miller and Ranum, discusses binary trees in Chapter 7.
- Elsewhere:
  - Cormen, Leiserson, Rivest, and Stein discusses graph theory and trees in Appendices B.4 and B.5, and binary search trees in Chapter 12.

REVISION HISTORY

- 2022-02-23 Last year's lecture on this topic finalized
- 2023-02-16 Updated for 2023