

LECTURE 13

MERGESORT

MCS 275 Spring 2023

Emily Dumas

LECTURE 13: MERGESORT

Reminders and announcements:

- Project 2 posted; due 6pm central Fri Feb 24.
- Project 1 grading underway.
- Homework 5 due tomorrow (notebook).

PROJECT 2

Demo and discussion to supplement the [project description](#).

PLAN

- Discuss the theory of
 - Divide and conquer
 - Sorting
 - Mergesort
- Implement mergesort

DIVIDE AND CONQUER

A strategy that often involves recursion.

- **Split** a problem into parts.
- **Solve** for each part.
- **Merge** the partial solutions into a solution of the original problem.

Not always possible or a good idea. It only works if merging partial solutions is easier than solving the entire problem.

COMPARISON SORT

Suppose you have a list of objects that can be compared with $==$, $>$, $<$.

You'd like to reorder them in increasing order.

This problem is called **comparison sort**. There are many solutions.

MERGESORT

A divide-and-conquer solution to comparison sort.

It is a fast solution, often used in practice.

Key: It is pretty easy to take two sorted lists and merge them into a single sorted list.

So, let's divide our list into halves, sort each one (recursively), then merge them.

Now we'll formalize this.

Algorithm `mergesort`:

Input: list \mathbb{L} whose elements support comparison.

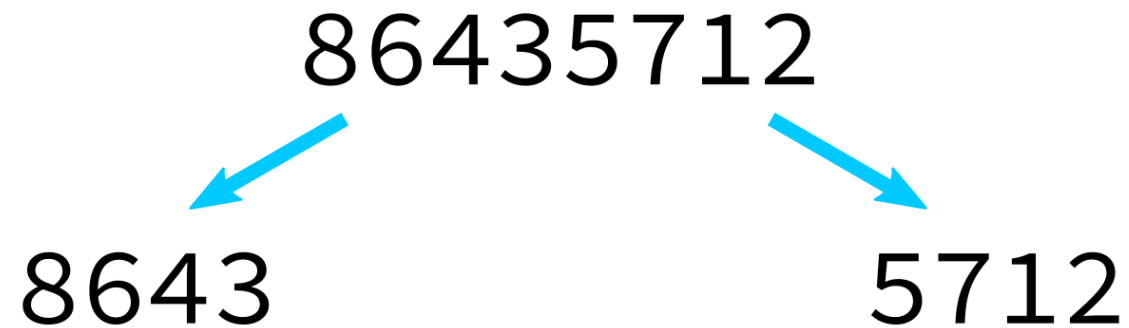
Goal: return a list that contains the items from \mathbb{L} but in sorted order.

1. If \mathbb{L} has 0 or 1 elements, return \mathbb{L}
2. Otherwise, divide \mathbb{L} into roughly equal pieces \mathbb{L}_0 and \mathbb{L}_1 .
3. Recursively call `mergesort` on \mathbb{L}_0 and \mathbb{L}_1 .
4. Use `merge` to merge these sorted lists and return the result.

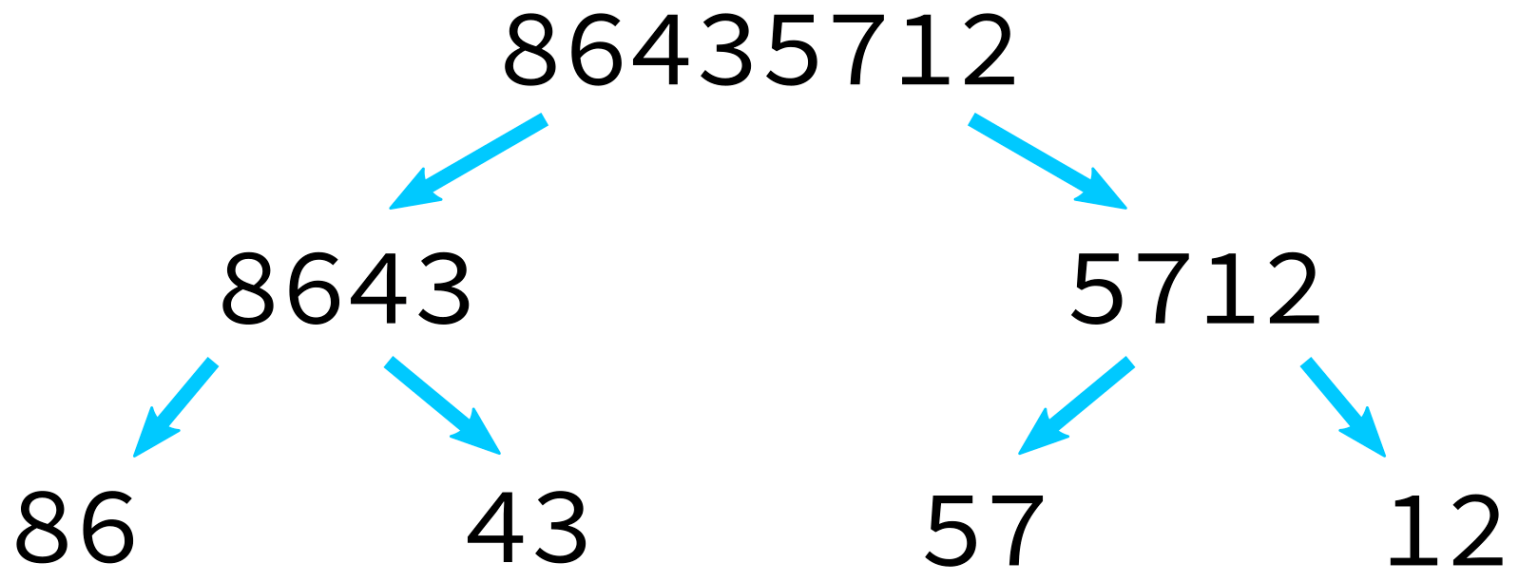
MERGESORT EXAMPLE

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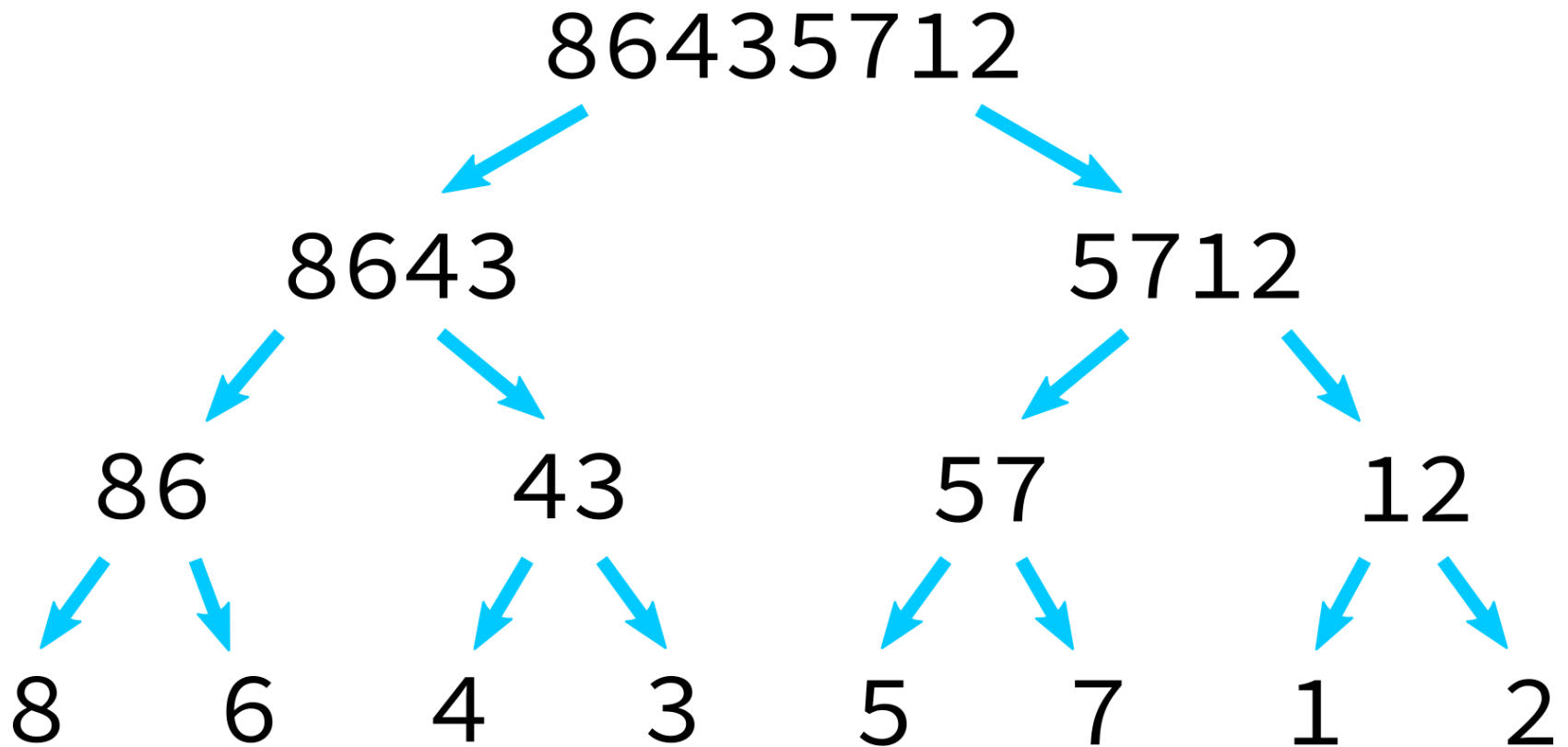
MERGESORT EXAMPLE



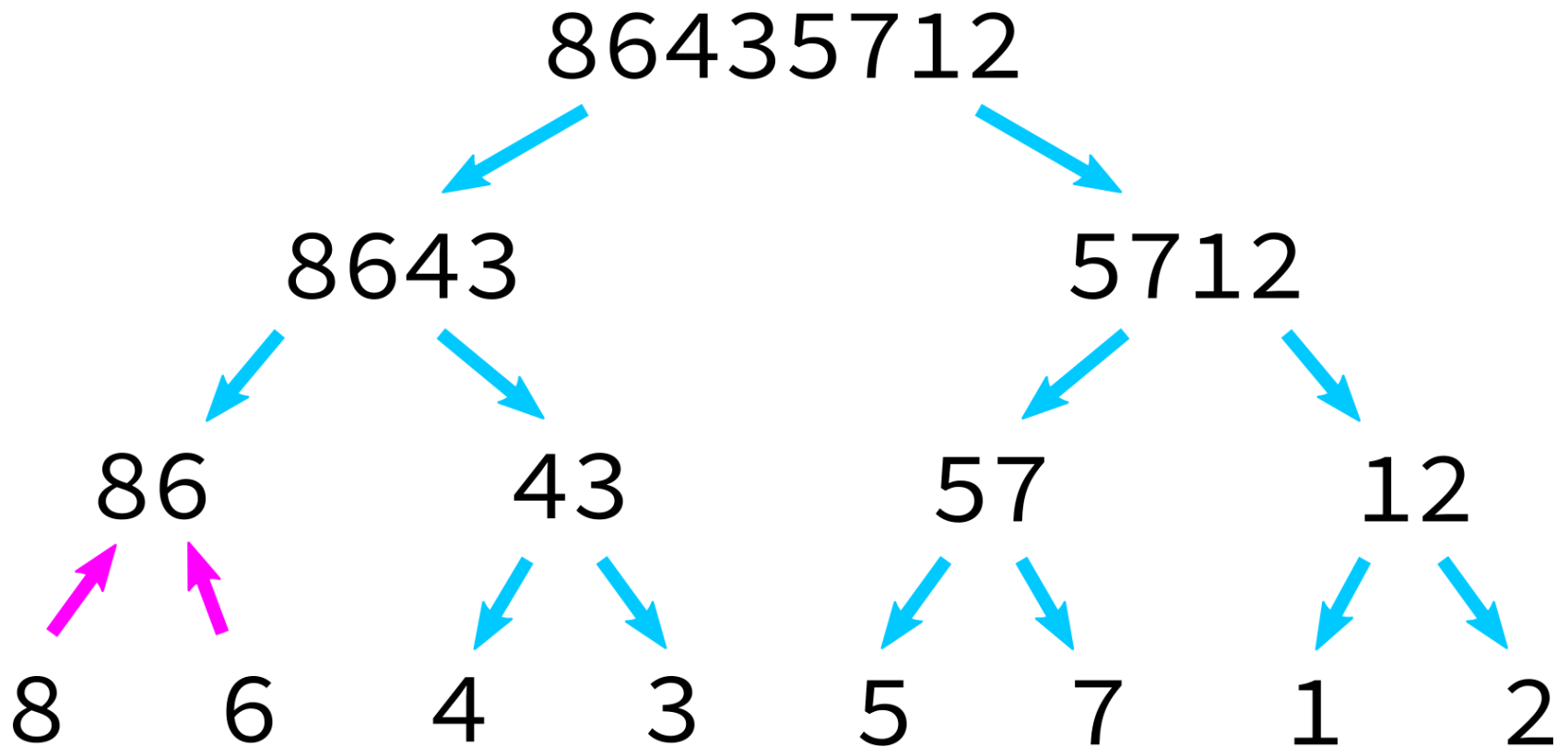
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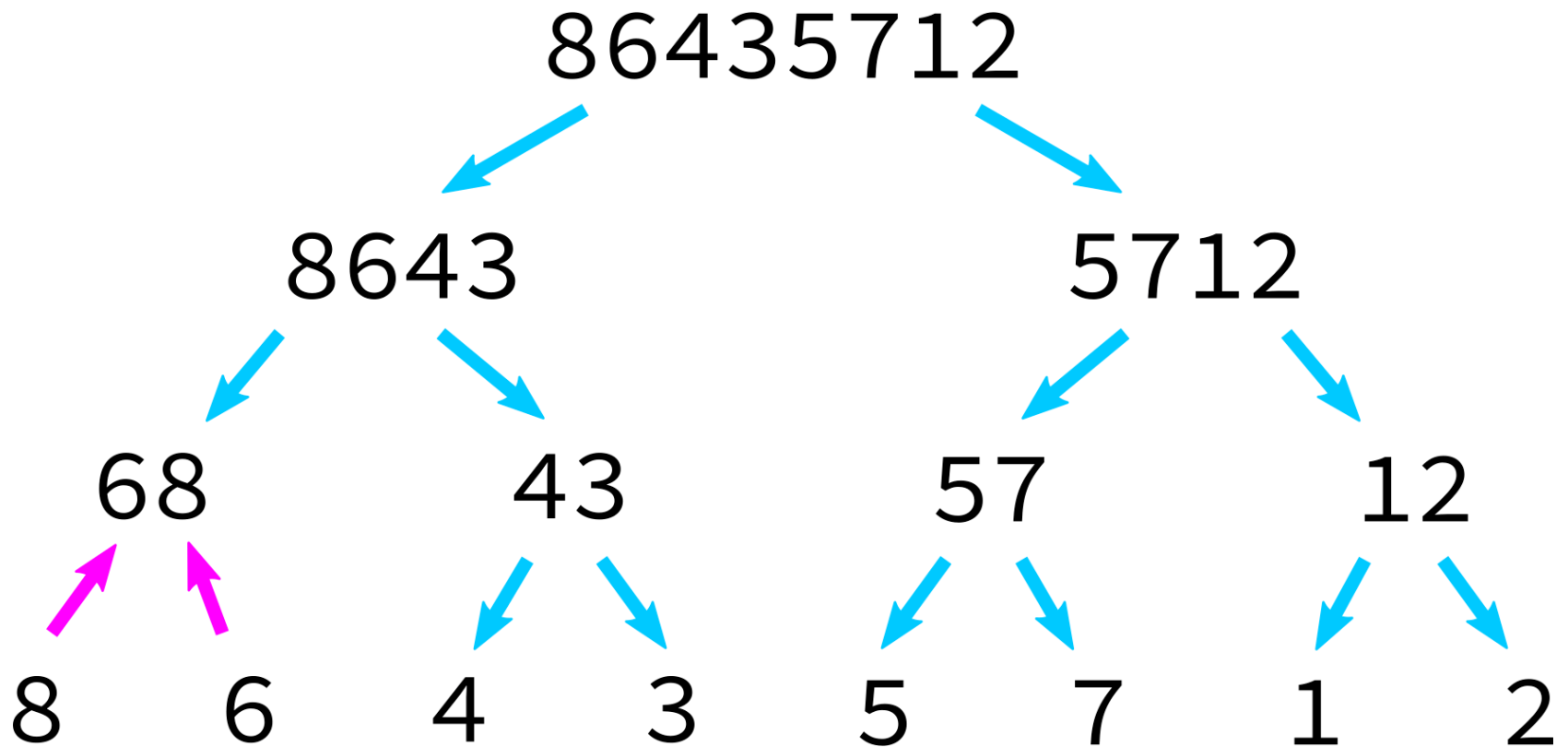
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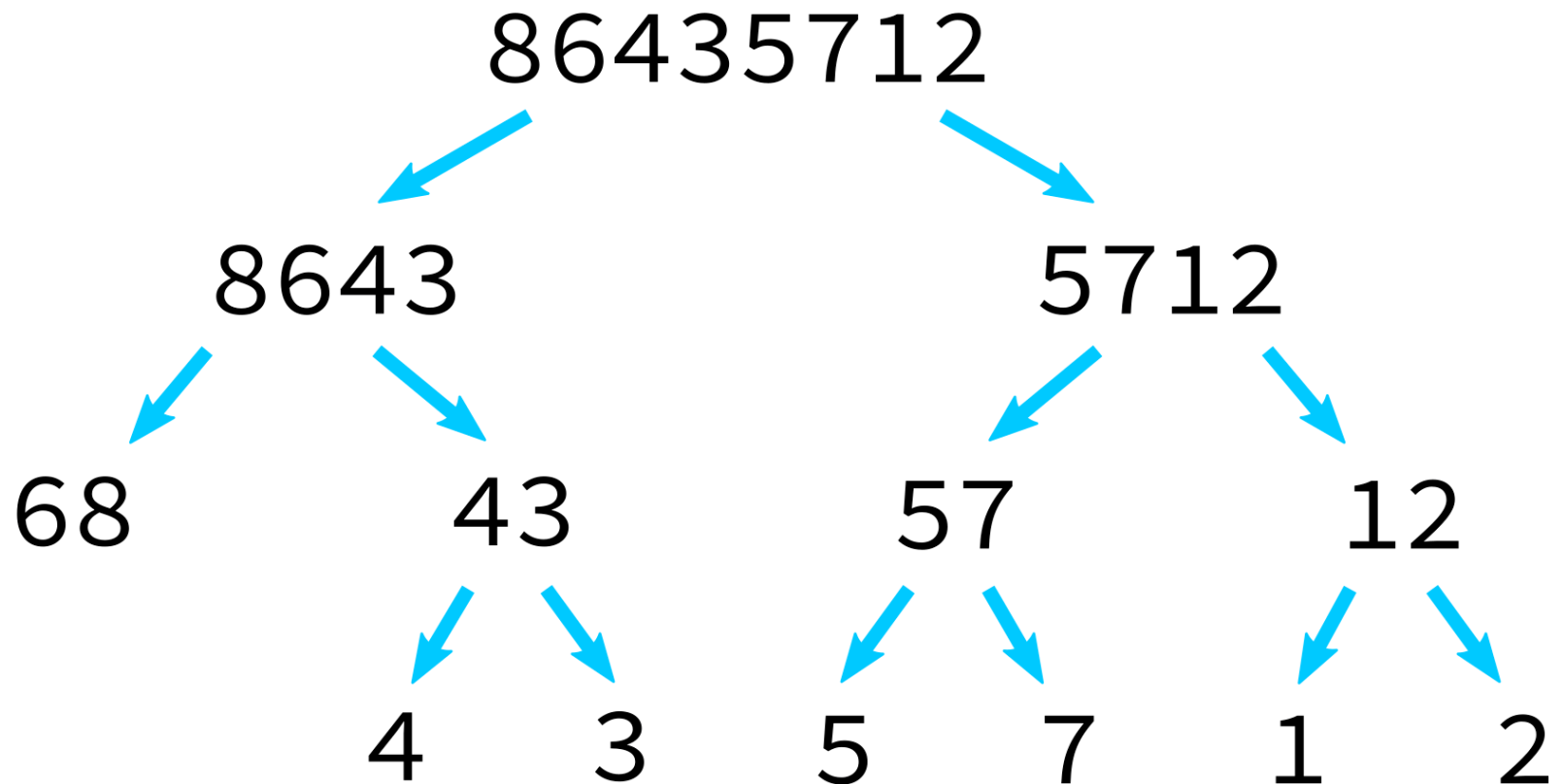
MERGESORT EXAMPLE



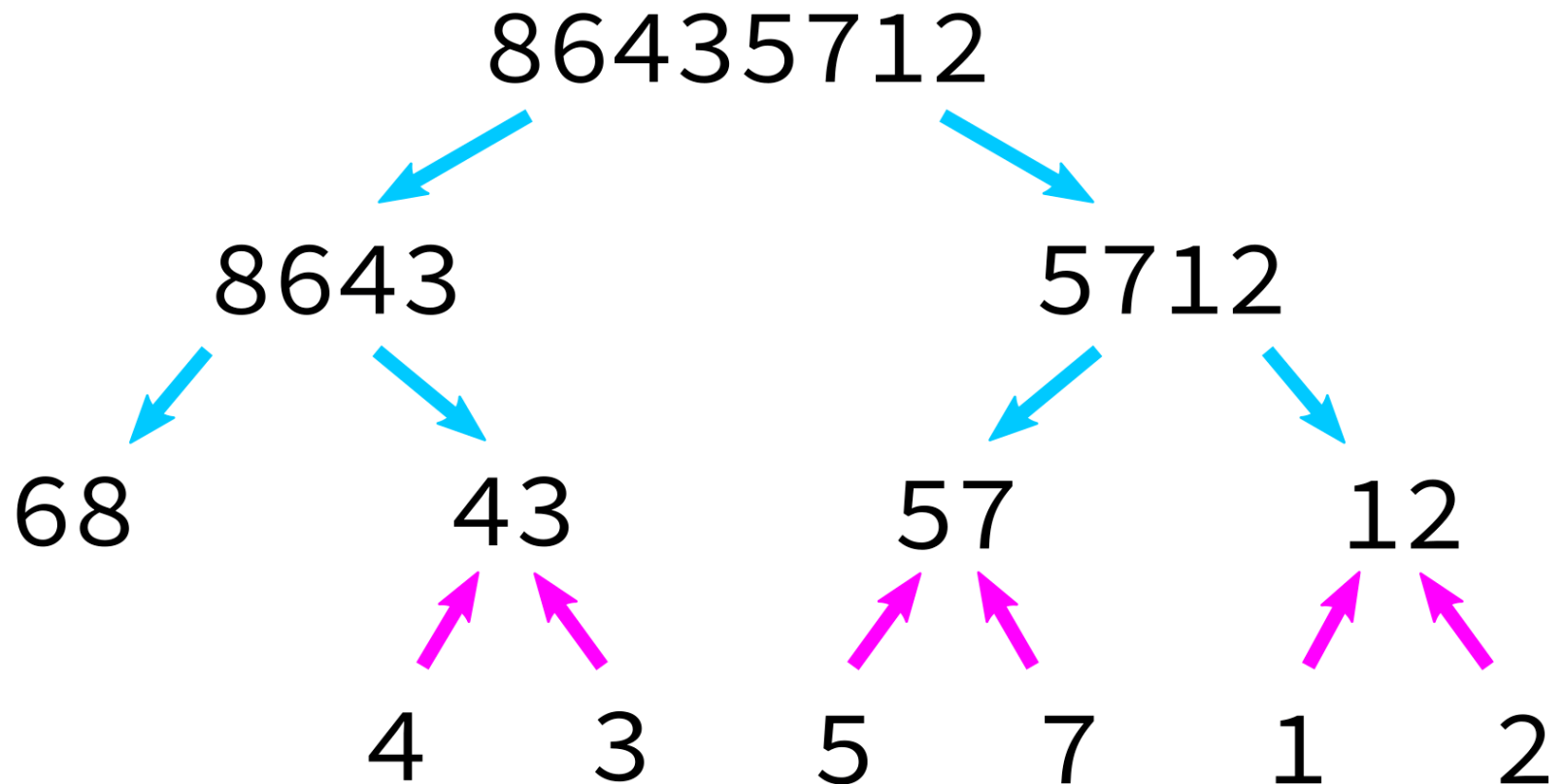
MERGESORT EXAMPLE



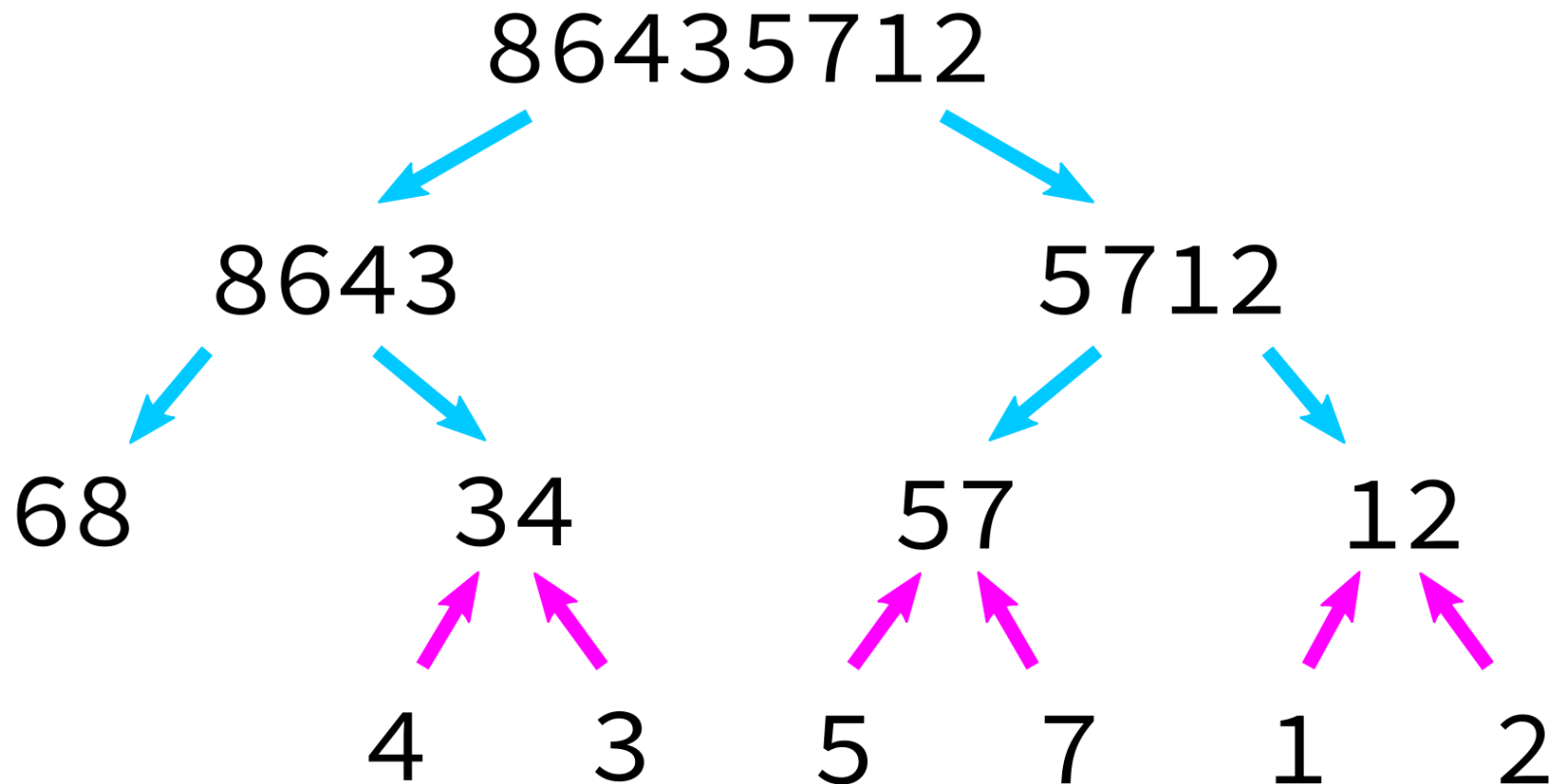
MERGESORT EXAMPLE



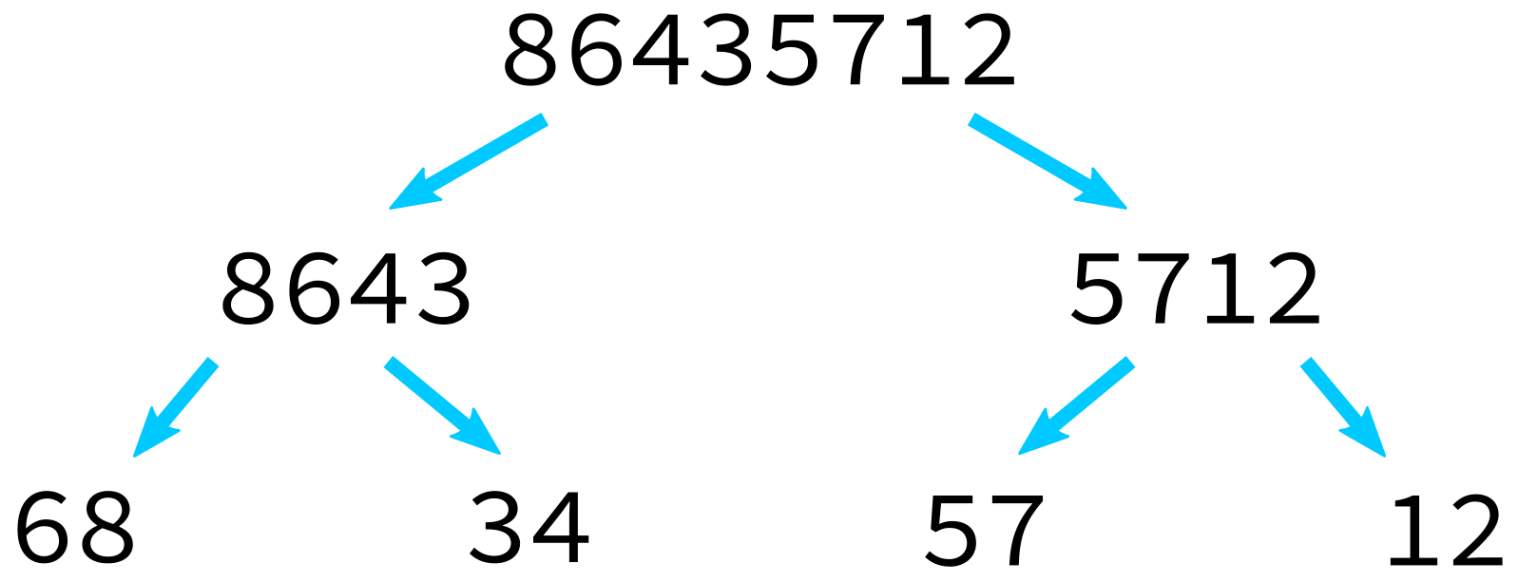
MERGESORT EXAMPLE



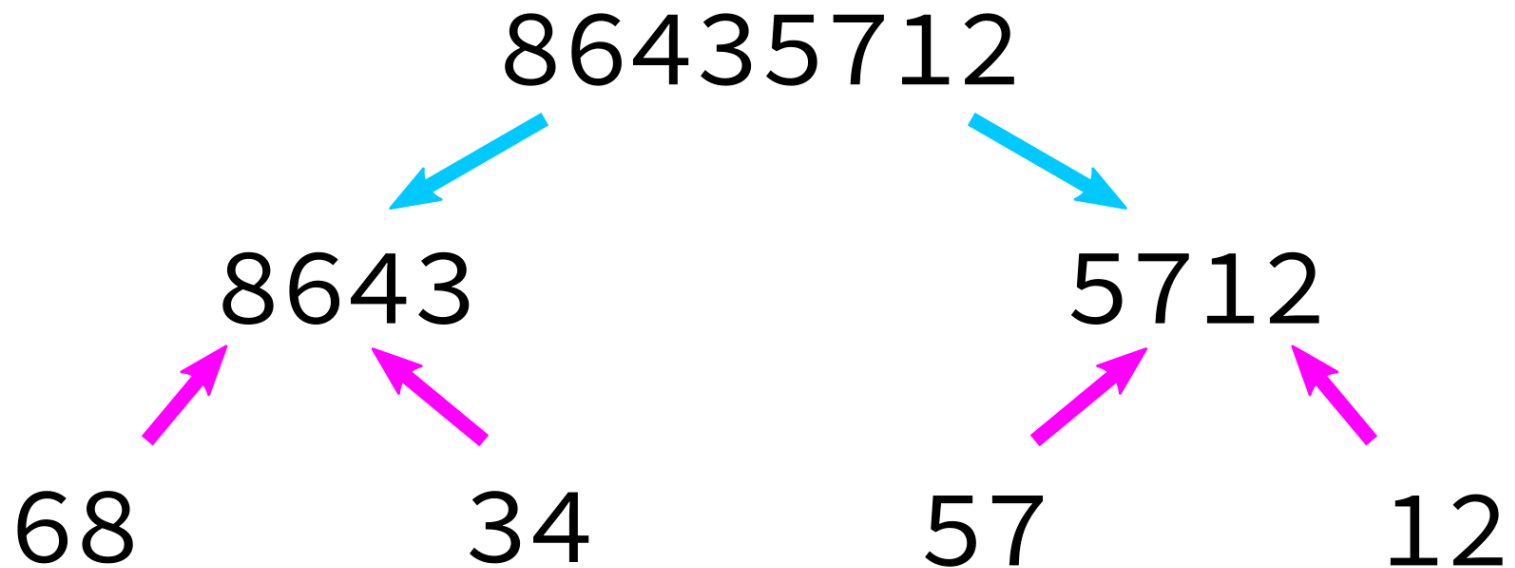
MERGESORT EXAMPLE



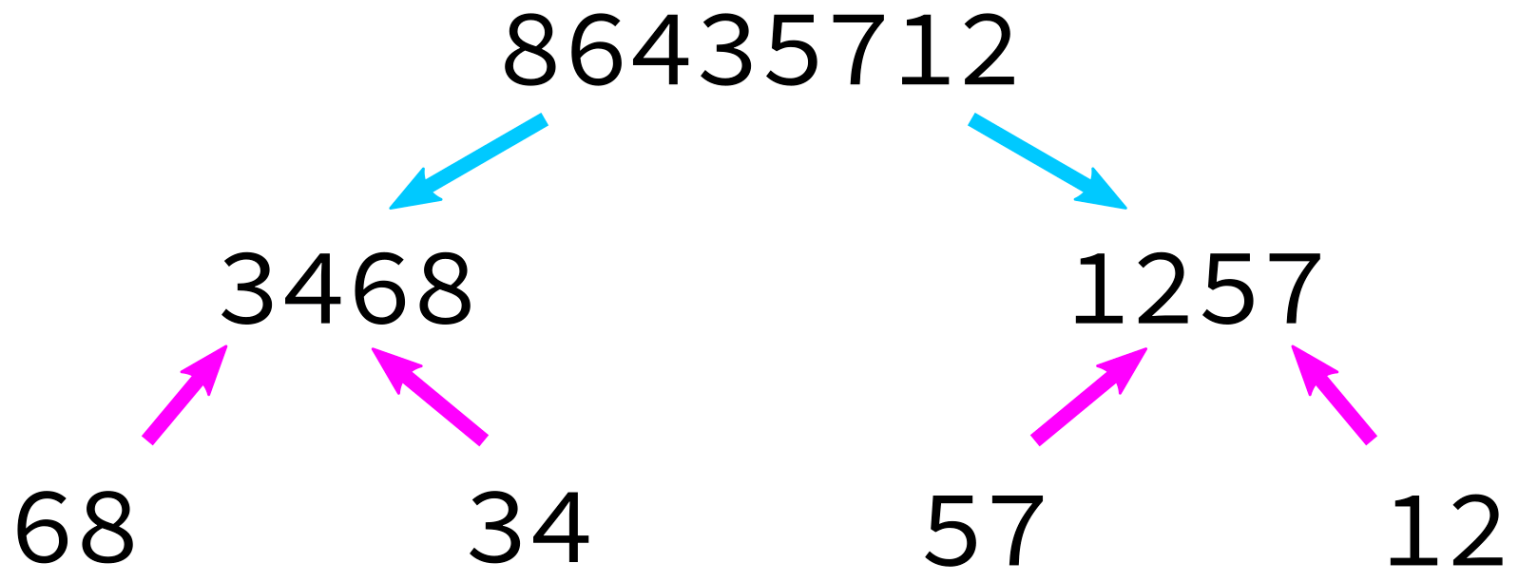
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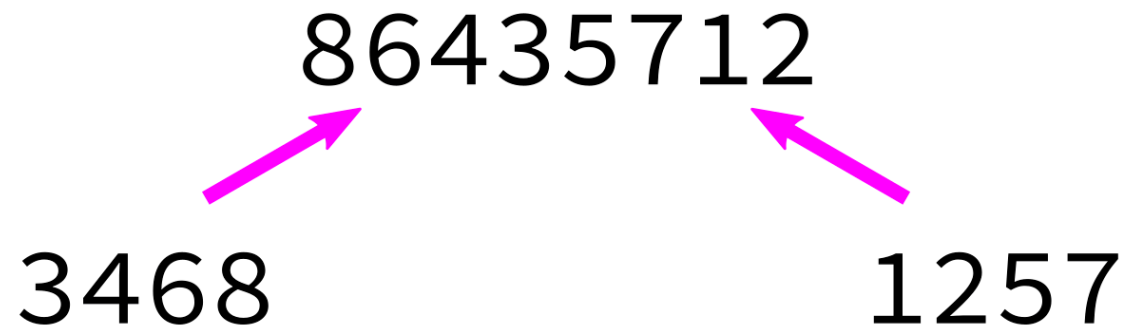
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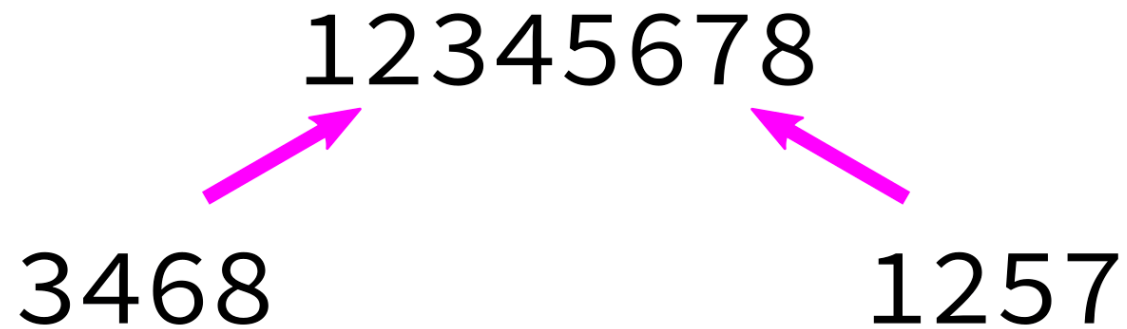
MERGESORT EXAMPLE



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MERGESORT EXAMPLE

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BUT HOW TO MERGE?

This algorithm depends on having a function `merge` that can merge two sorted lists into a single sorted list.

Algorithm merge:

Input: sorted lists L_0 and L_1 .

Goal: return a sorted list with same items as $L_0 + L_1$

1. Make a new empty list L
2. Make integer variables i_0, i_1 to keep track of current position in L_0, L_1 respectively. Set to zero.
3. While $i_0 < \text{len}(L_0)$ and $i_1 < \text{len}(L_1)$, do the following:
 - Check which of $L_0[i_0]$ and $L_1[i_1]$ is smaller.
 - Append the smaller one to L .
 - Increment whichever one of i_0, i_1 was used.
4. Append any remaining portion of L_0 to L .
5. Append any remaining portion of L_1 to L .

MERGING SORTED LISTS

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MERGING SORTED LISTS

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MERGING SORTED LISTS

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CODING TIME

Let's implement `mergesort` in Python.

REFERENCES

- Recursion references from [Lecture 10](#).
- Making nice visualizations of sorting algorithms is a cottage industry in CS education. Some you might like to check out:
 - [2D visualization through color sorting](#) by Linus Lee
 - [Animated bar graph visualization of many sorting algorithms](#) by Alex Macy
 - Slanted line animated visualizations of [mergesort](#) and [quicksort](#) by Mike Bostock

REVISION HISTORY

- 2022-02-16 Last year's version of this lecture finalized
- 2023-02-13 Updated for 2023

