LECTURE 13

MERGESORT

MCS 275 Spring 2023

Emily Dumas

LECTURE 13: MERGESORT

Reminders and announcements:

- Project 2 posted; due 6pm central Fri Feb 24.
- Project 1 grading underway.
- Homework 5 due tomorrow (notebook).

PROJECT 2

Demo and discussion to supplement the project description.

PLAN

- Discuss the theory of
  - Divide and conquer
  - Sorting
  - Mergesort
- Implement mergesort

DIVIDE AND CONQUER

A strategy that often involves recursion.

- **Split** a problem into parts.
- **Solve** for each part.
- **Merge** the partial solutions into a solution of the original problem.

Not always possible or a good idea. It only works if merging partial solutions is easier than solving the entire problem.

COMPARISON SORT

Suppose you have a list of objects that can be compared with $==$, $>$, $<$.

You'd like to reorder them in increasing order.

This problem is called **comparison sort**. There are many solutions.

MERGESORT

A divide-and-conquer solution to comparison sort.

It is a fast solution, often used in practice.

Key: It is pretty easy to take two sorted lists and merge them into a single sorted list.

So, let's divide our list into halves, sort each one (recursively), then merge them.

Now we'll formalize this.

Algorithm `mergesort`:

Input: list `L` whose elements support comparison.

Goal: return a list that contains the items from `L` but in sorted order.

1. If `L` has 0 or 1 elements, return `L`
2. Otherwise, divide `L` into rougly equal pieces `L0` and `L1`.
3. Recursively call `mergesort` on `L0` and `L1`.
4. Use `merge` to merge these sorted lists and return the result.

MERGESORT EXAMPLE

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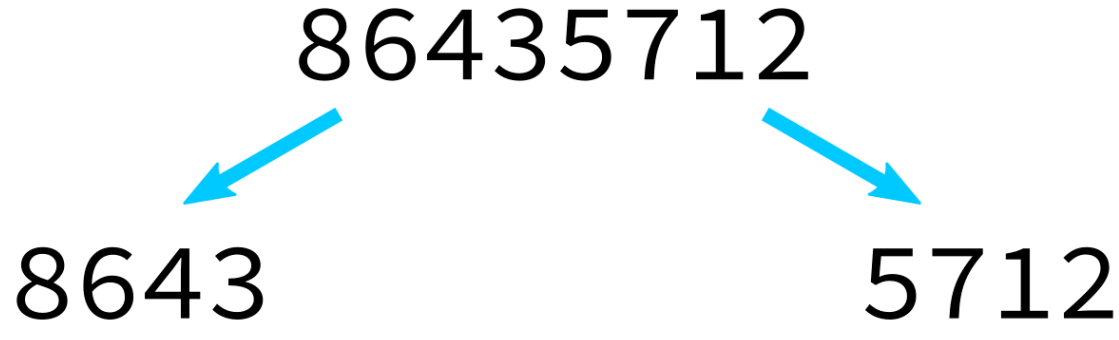
MERGESORT EXAMPLE



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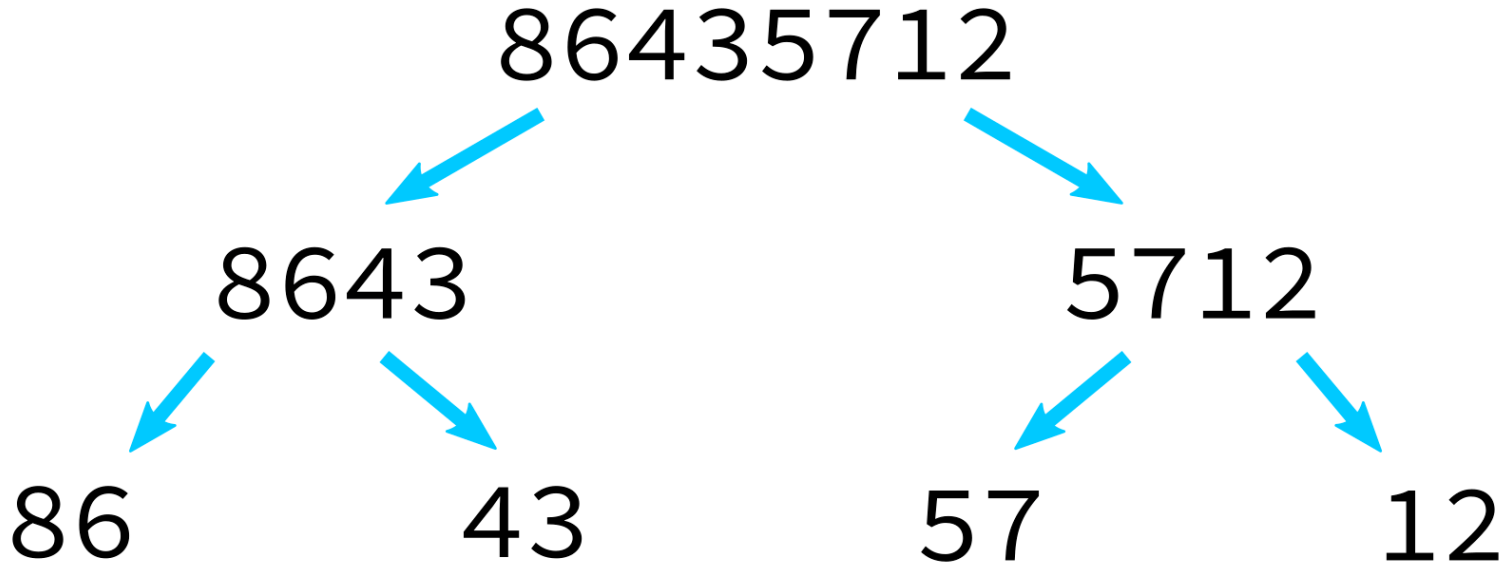

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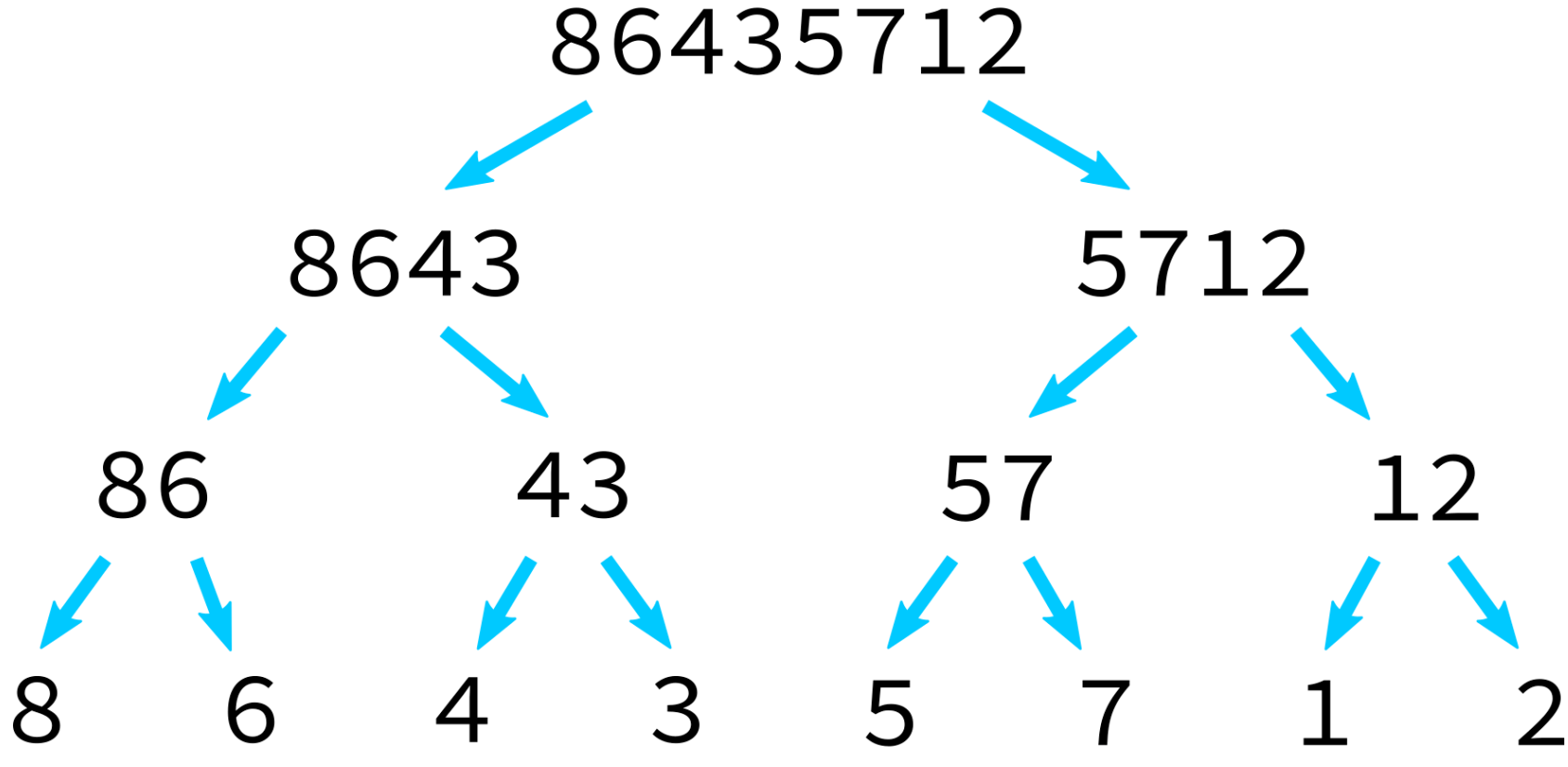

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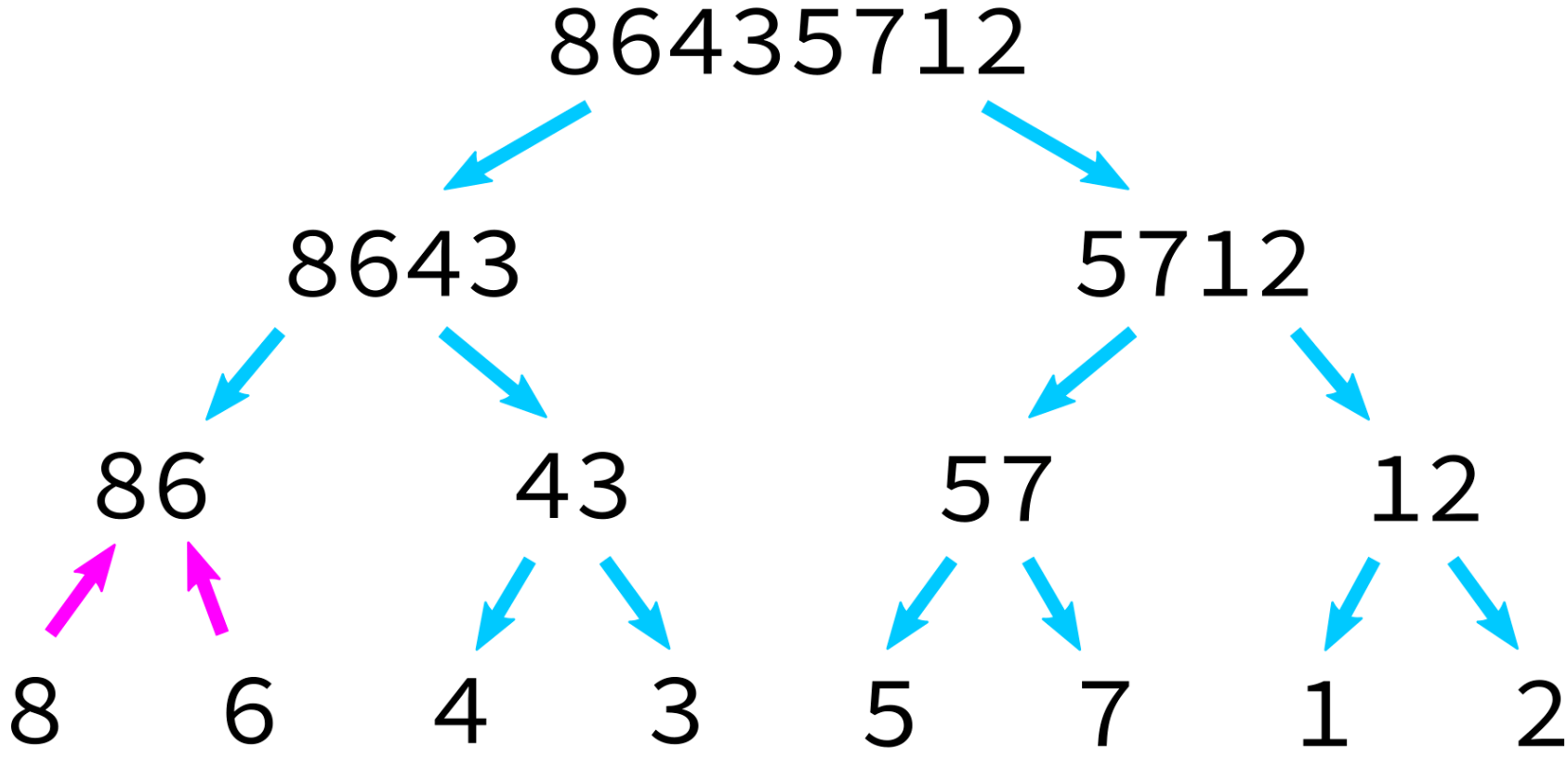

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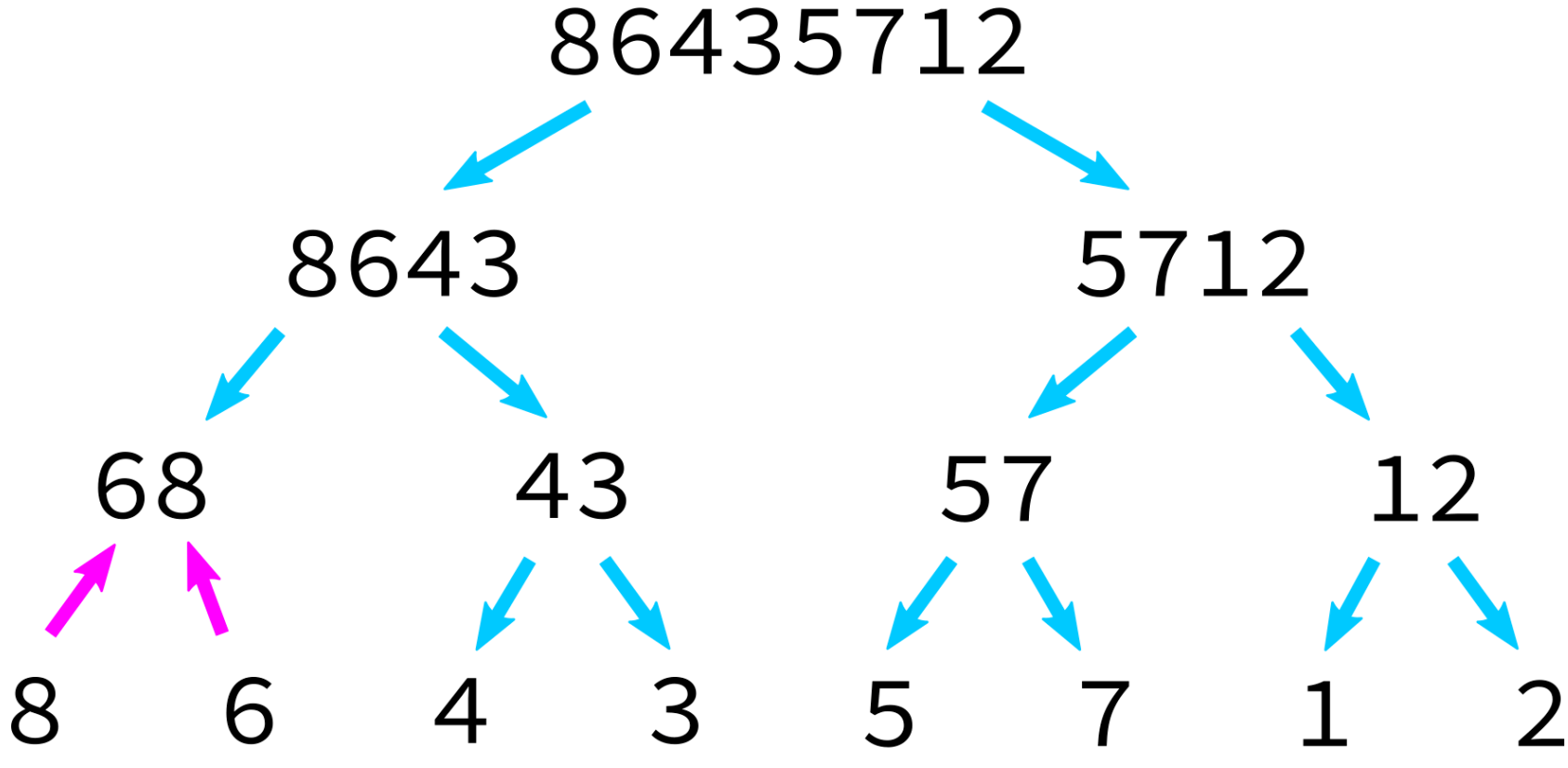

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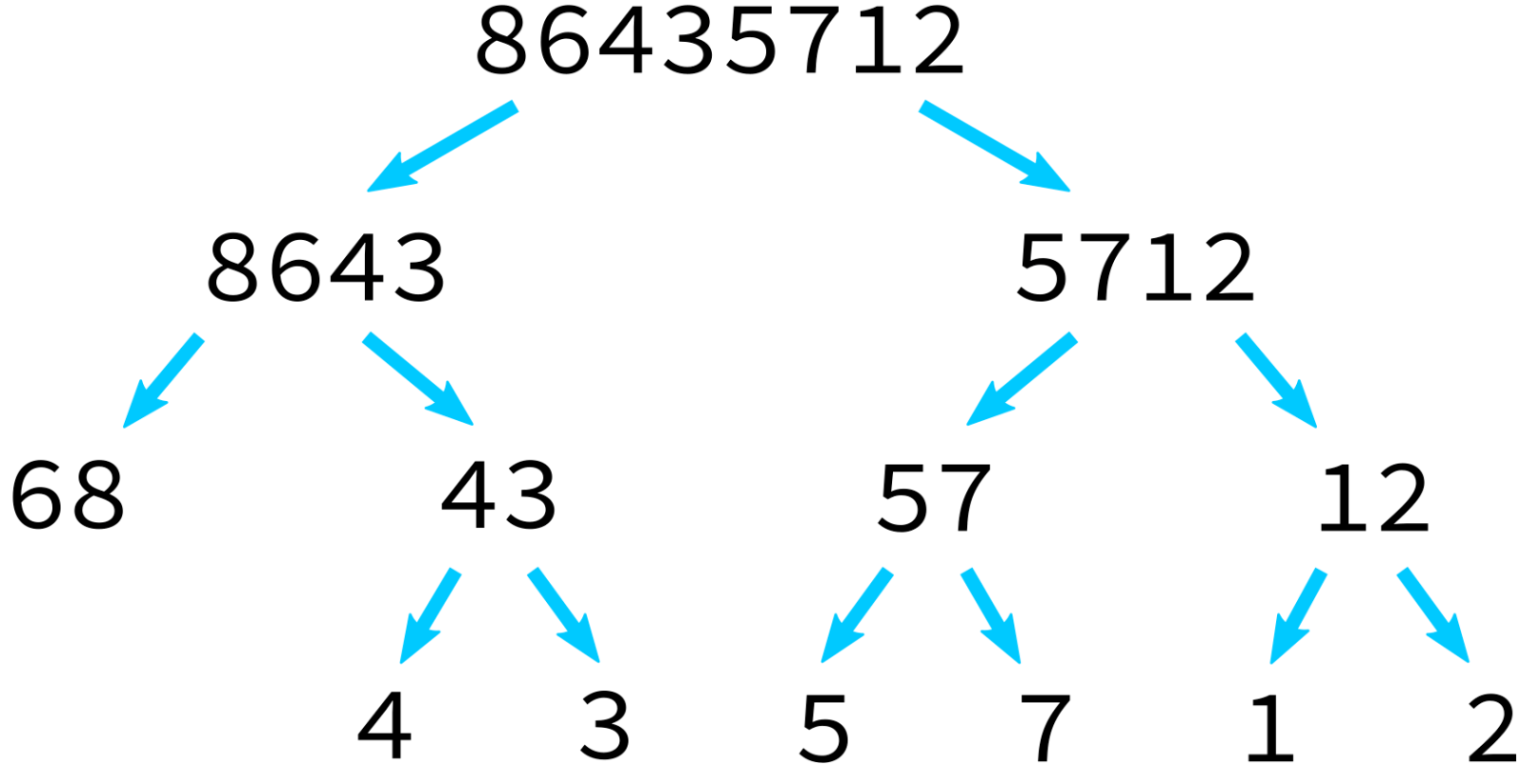

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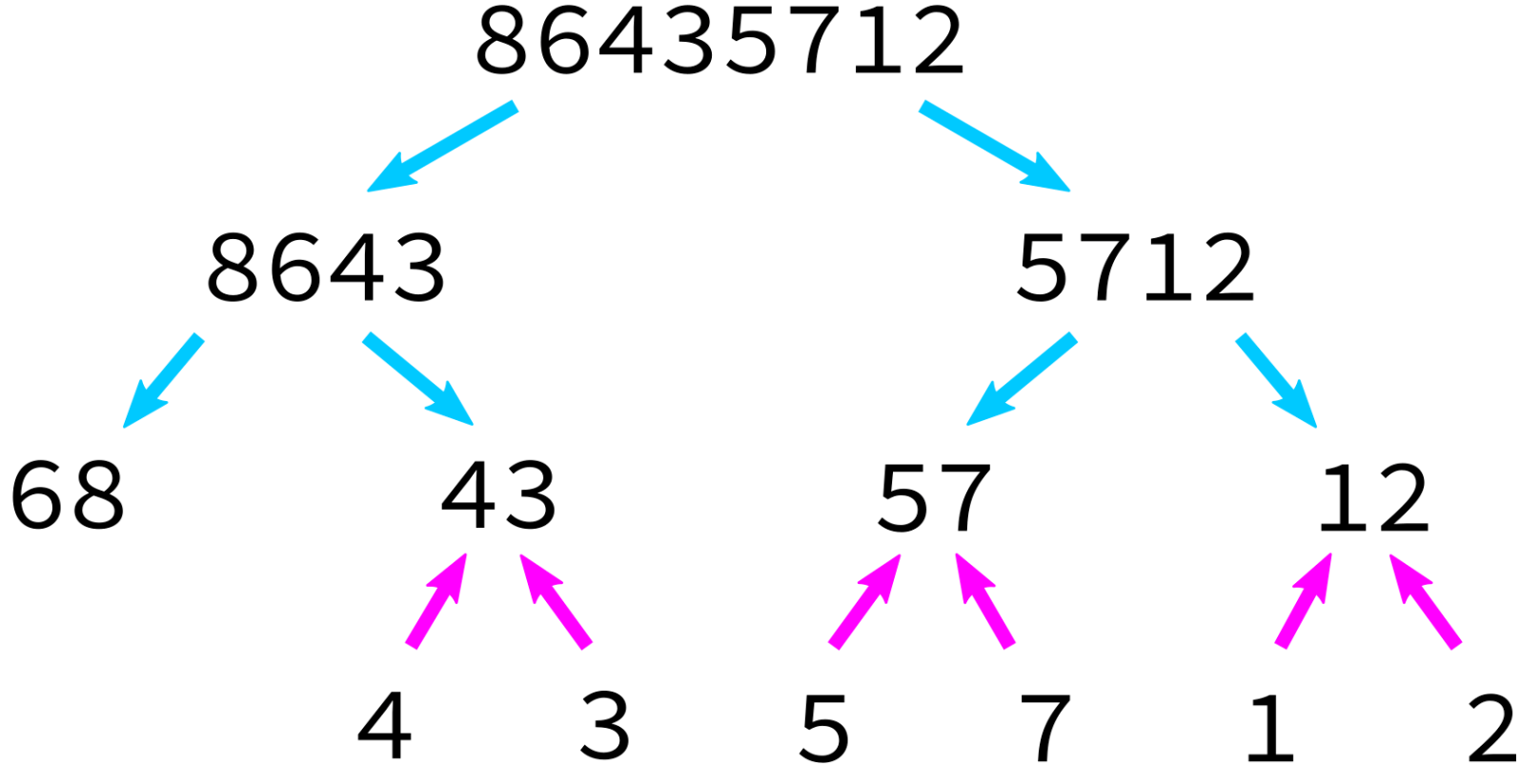

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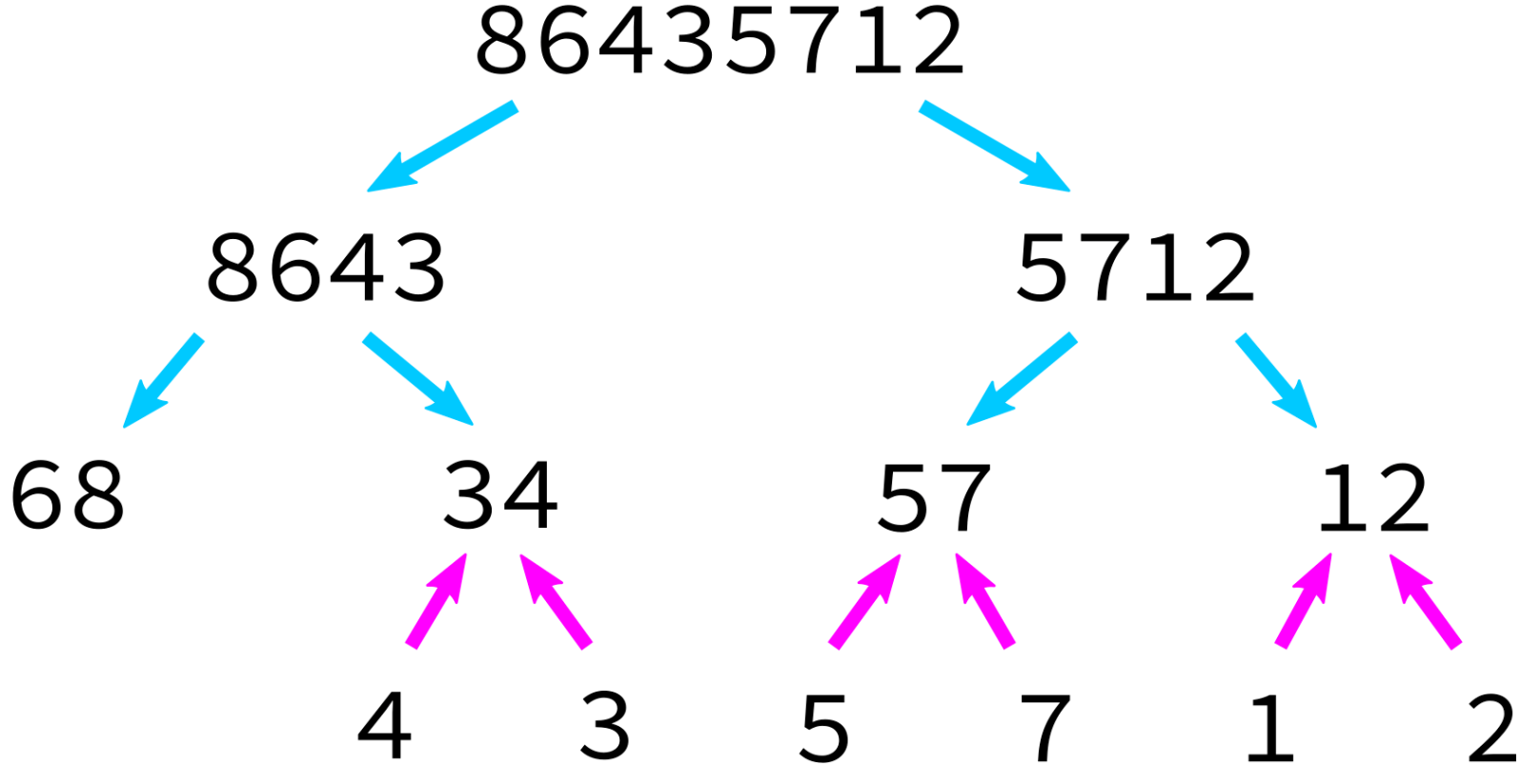

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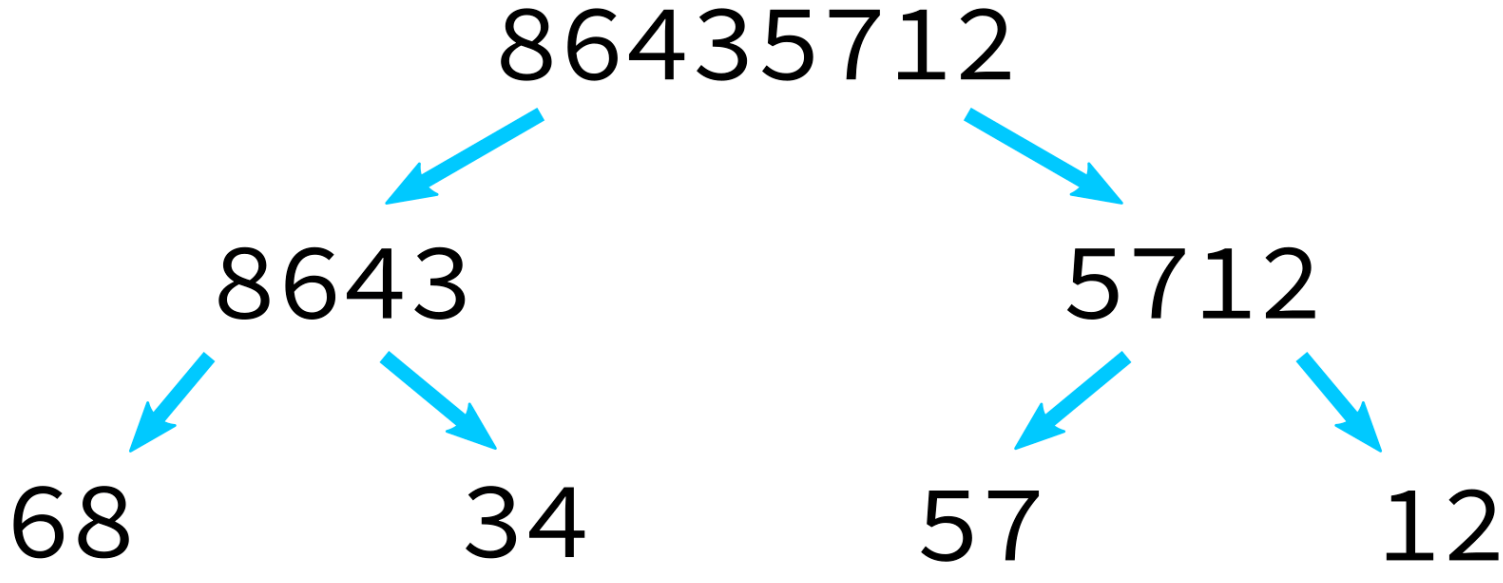

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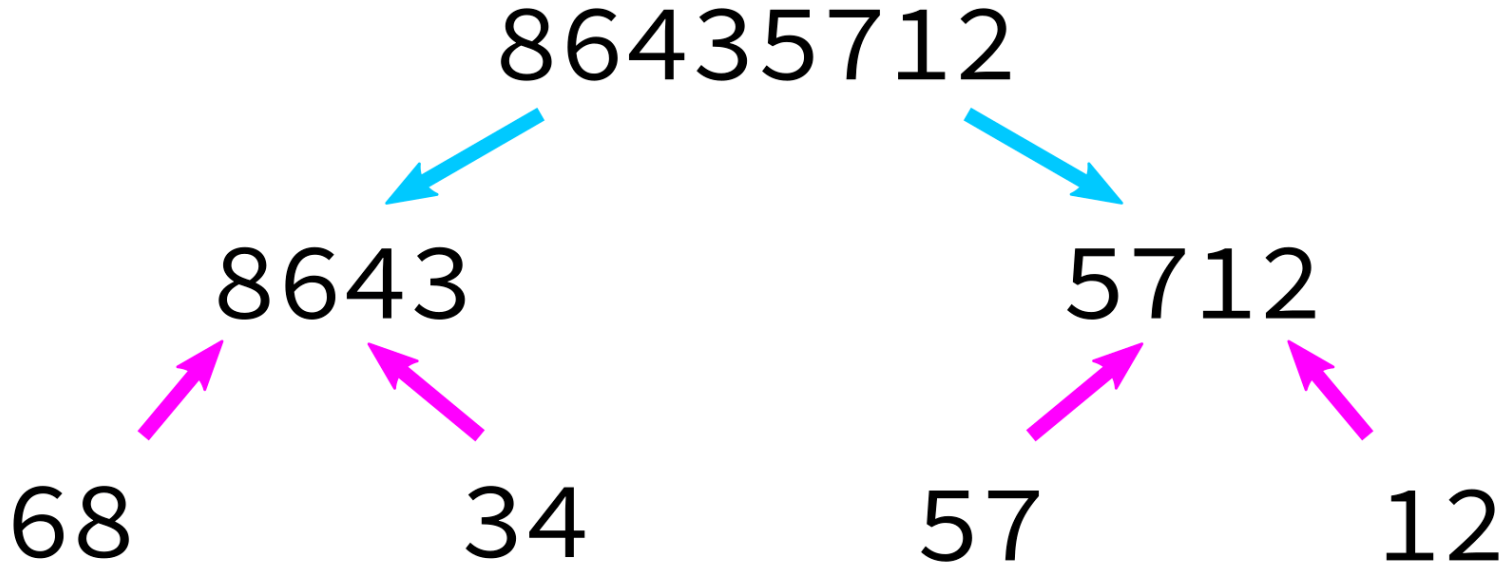

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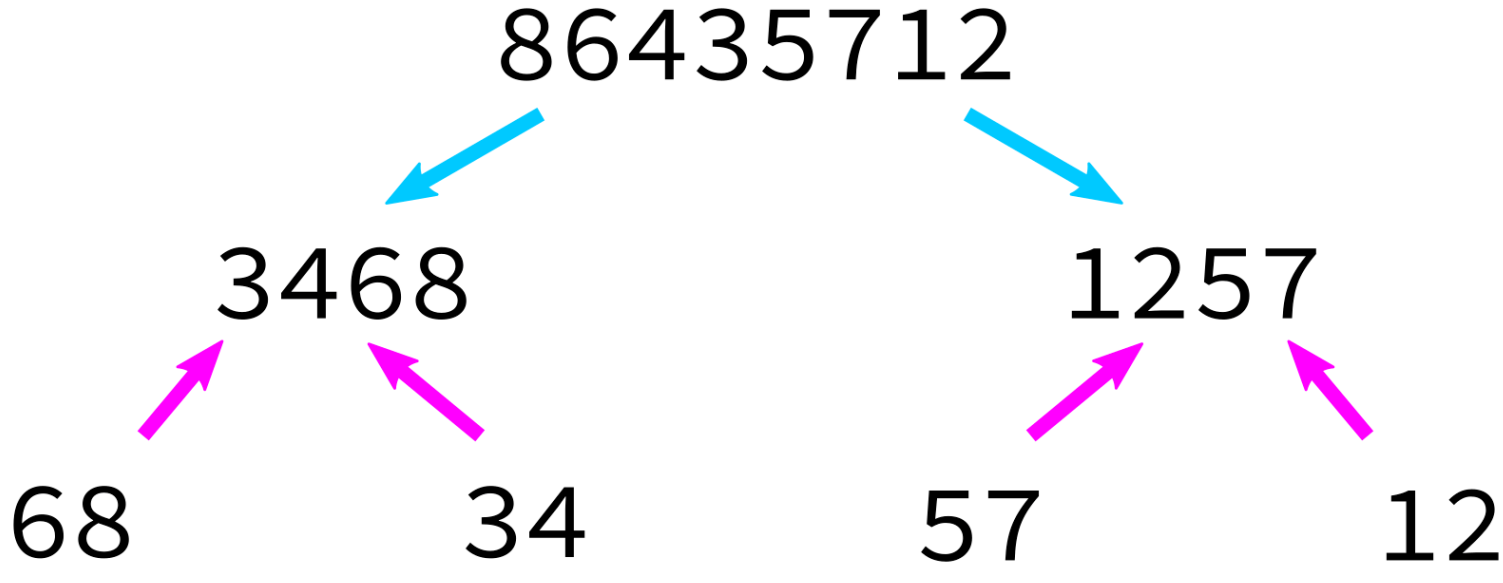

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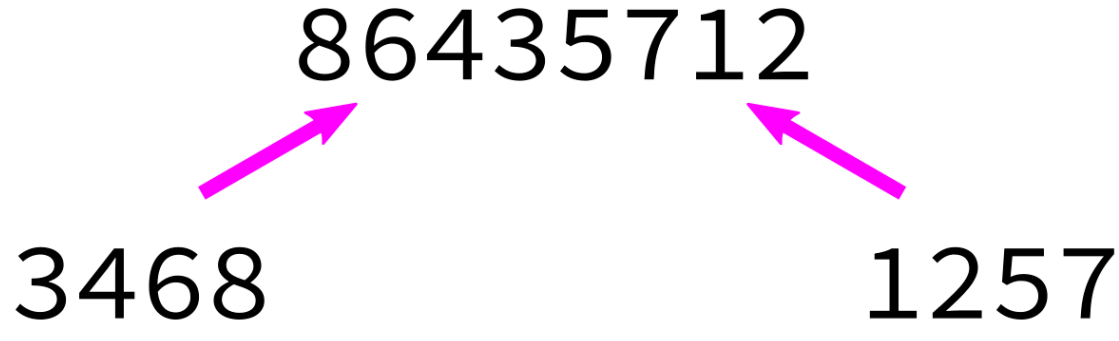

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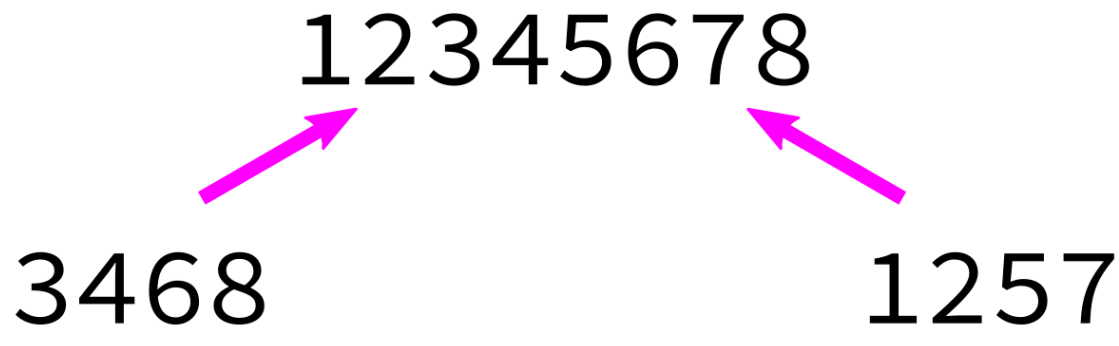

MERGESORT EXAMPLE

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BUT HOW TO MERGE?

This algorithm depends on having a function `merge` that can merge two sorted lists into a single sorted list.

Algorithm `merge`:

Input: sorted lists `L0` and `L1`.

Goal: return a sorted list with same items as `L0+L1`

1. Make a new empty list `L`
2. Make integer variables `i0, i1` to keep track of current position in `L0, L1` respectively. Set to zero.
3. While `i0 < len(L0)` and `i1 < len(L1)`, do the following:
   - Check which of `L0[i0]` and `L1[i1]` is smaller.
   - Append the smaller one to `L`.
   - Increment whichever one of `i0, i1` was used.
4. Append any remaining portion of `L0` to `L`.
5. Append any remaining portion of `L1` to `L`.

MERGING SORTED LISTS

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MERGING SORTED LISTS

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MERGING SORTED LISTS

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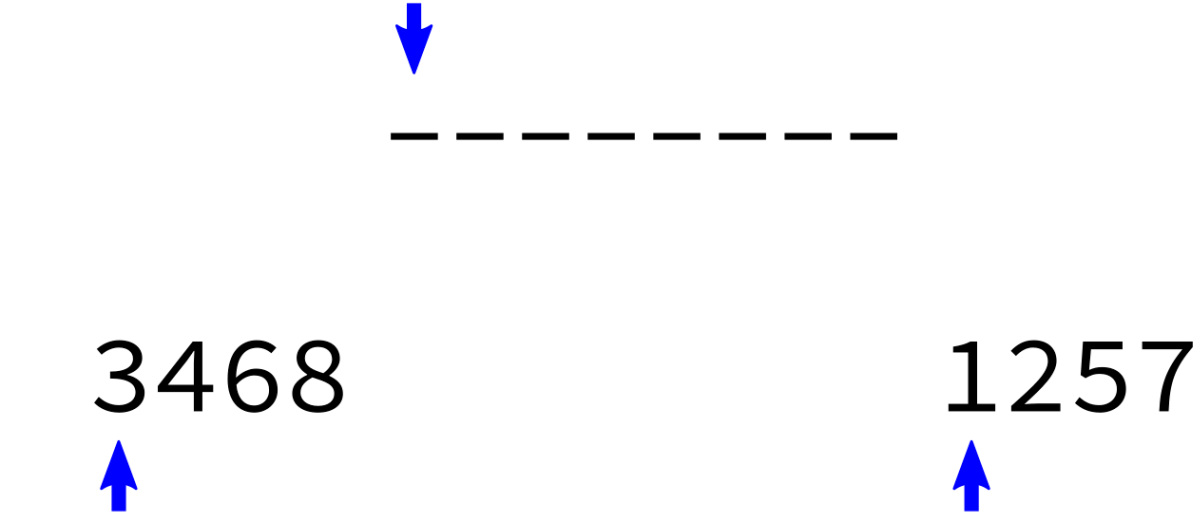

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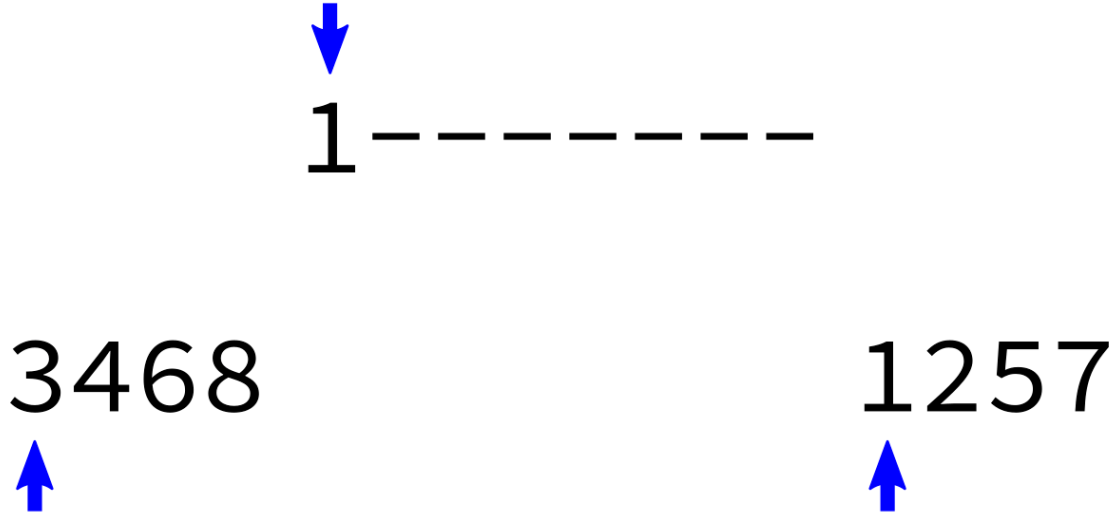

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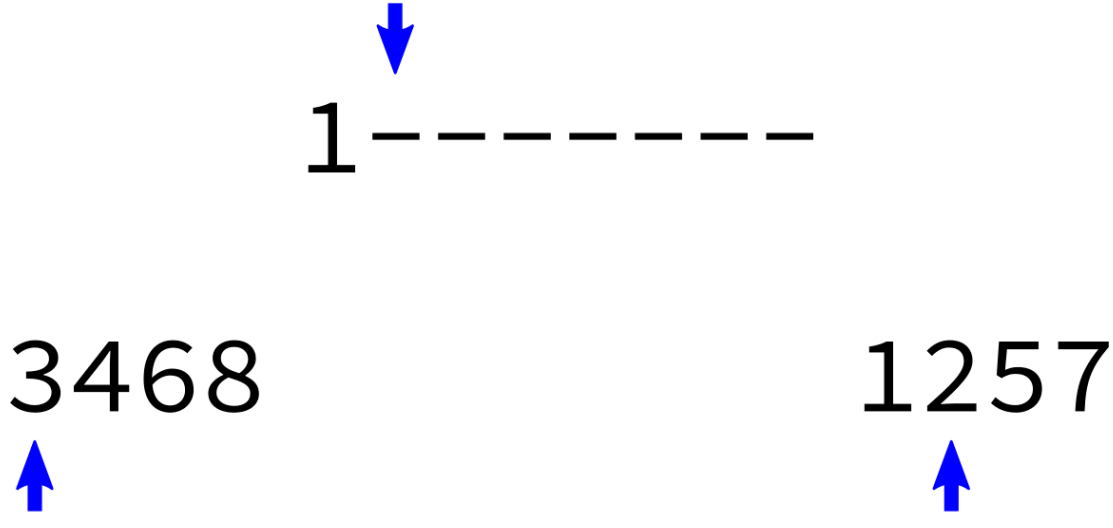

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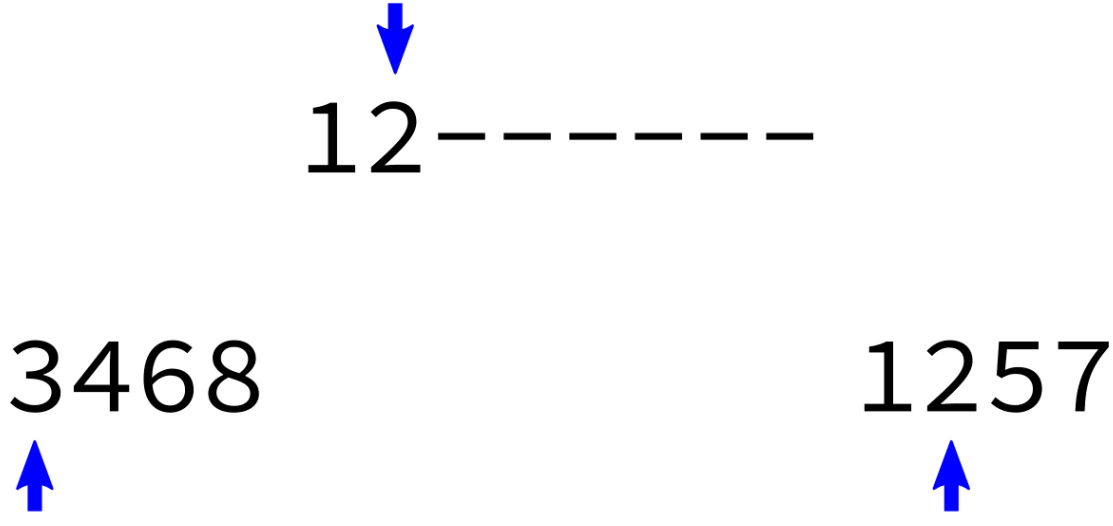

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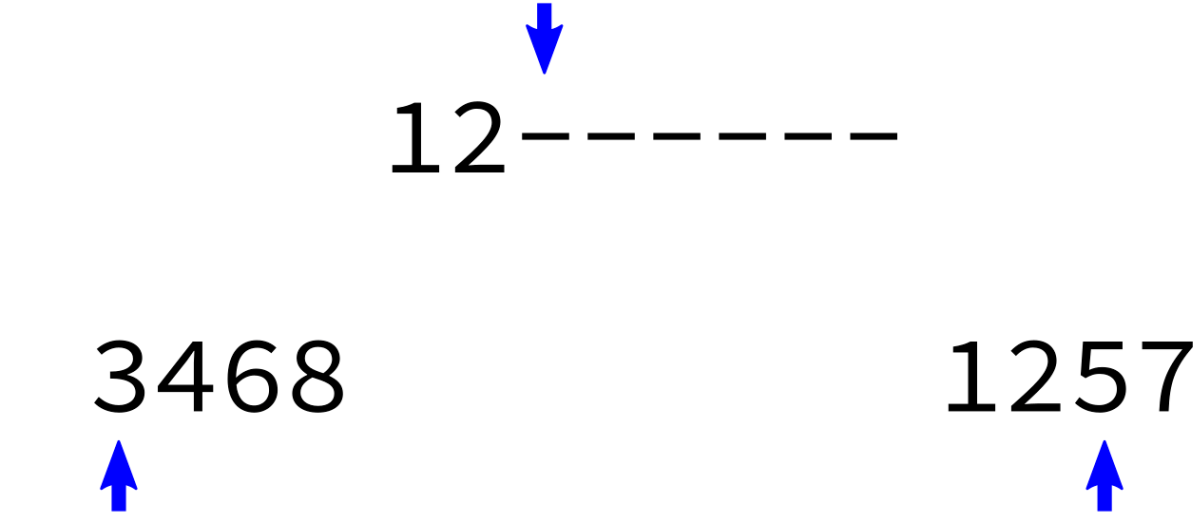

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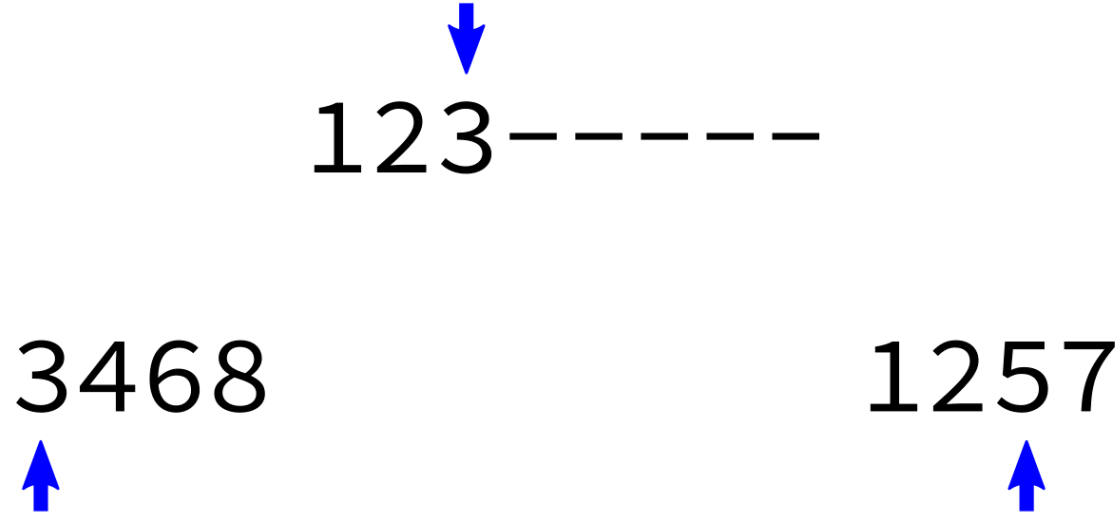

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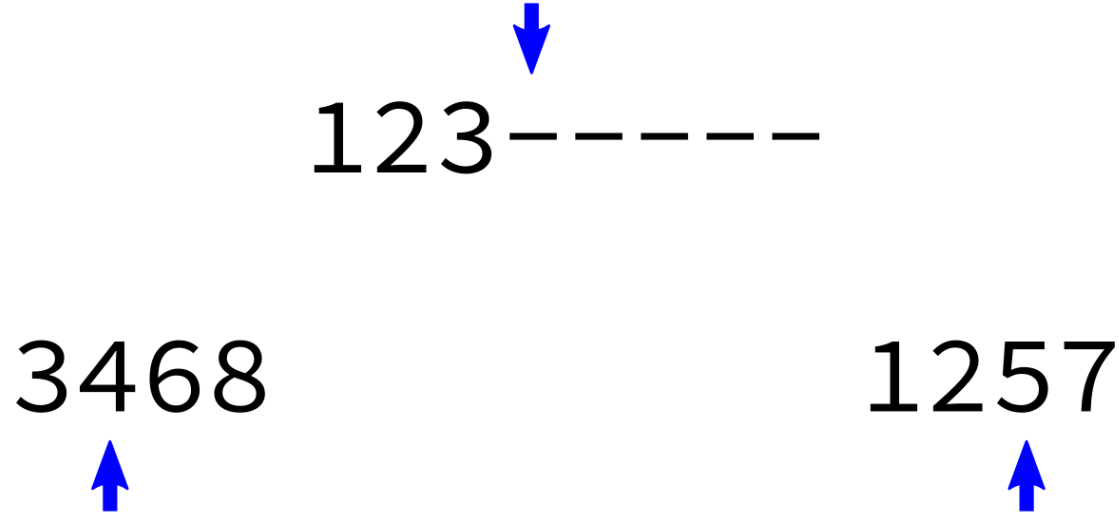

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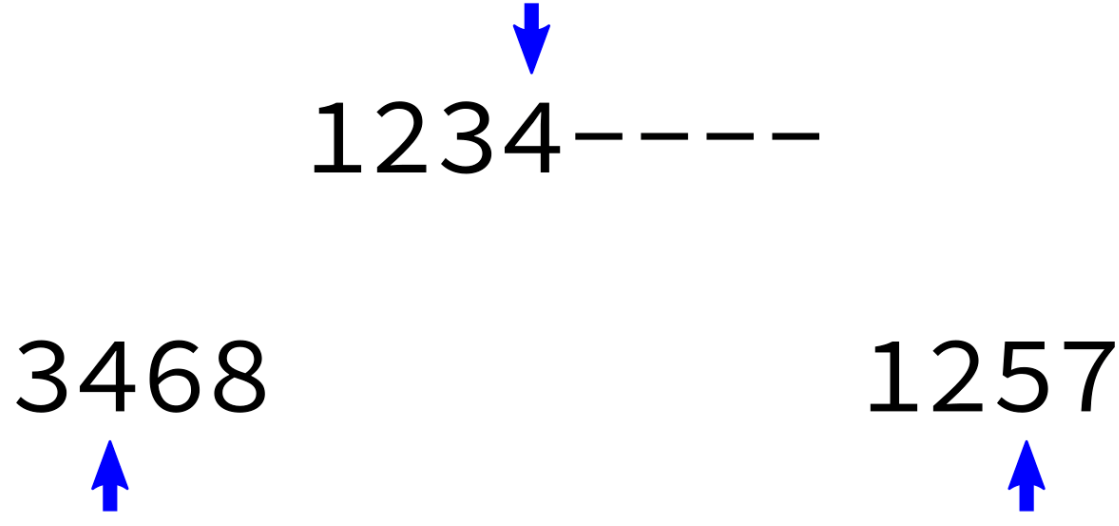

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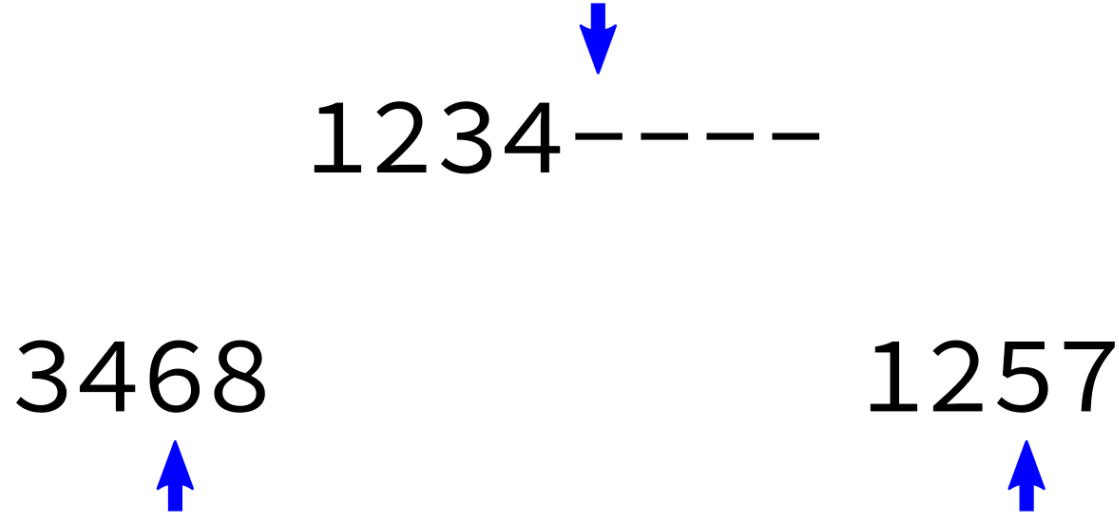

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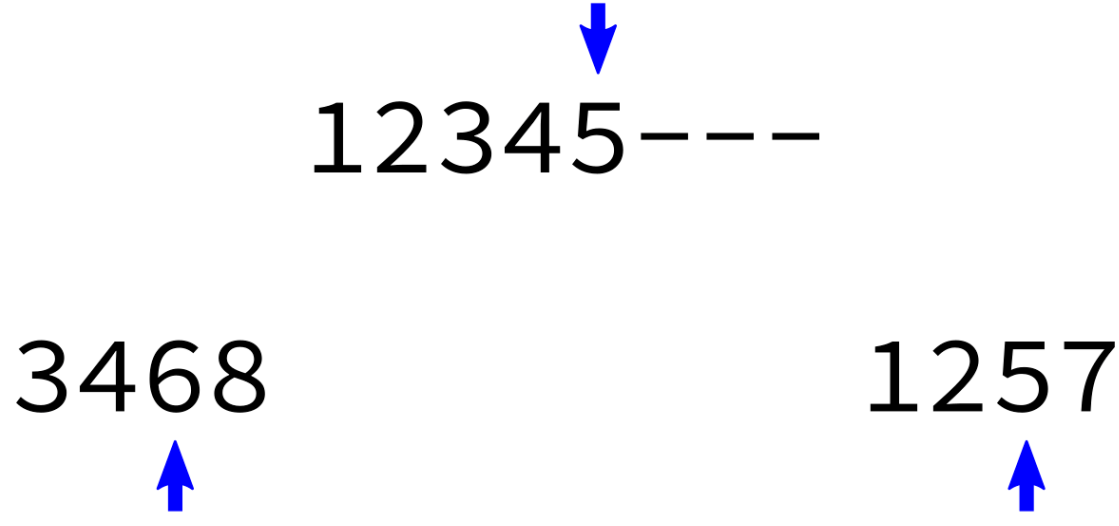

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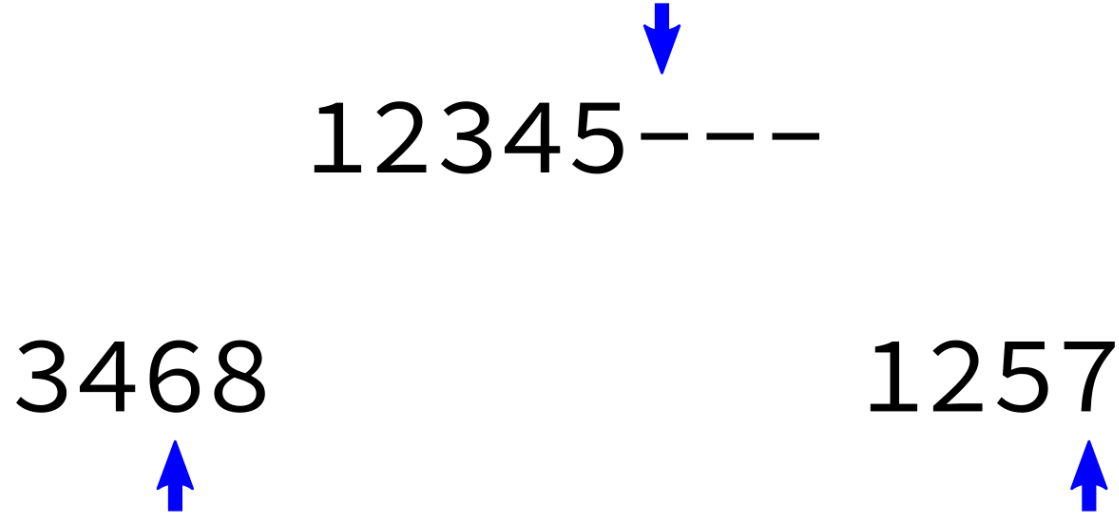

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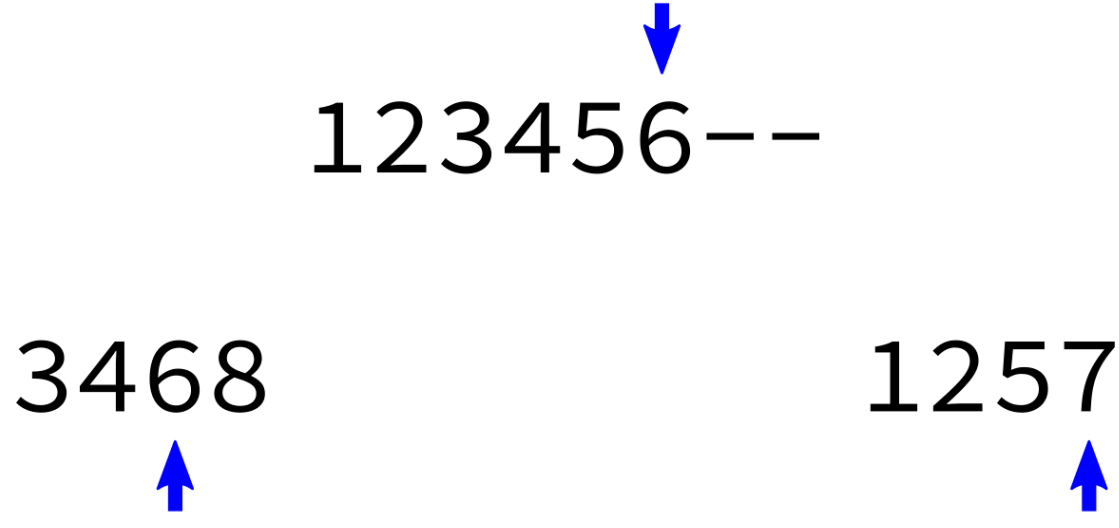

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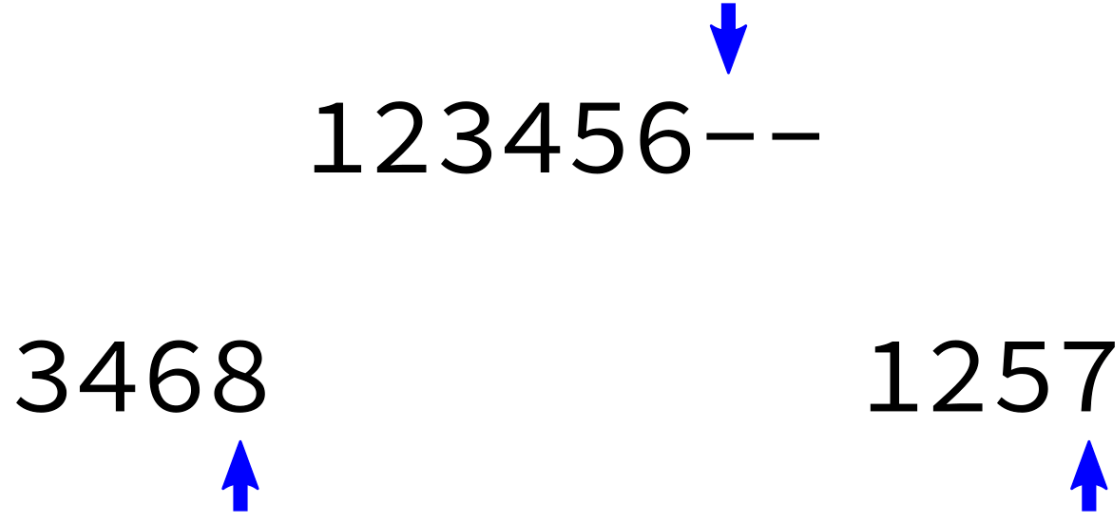

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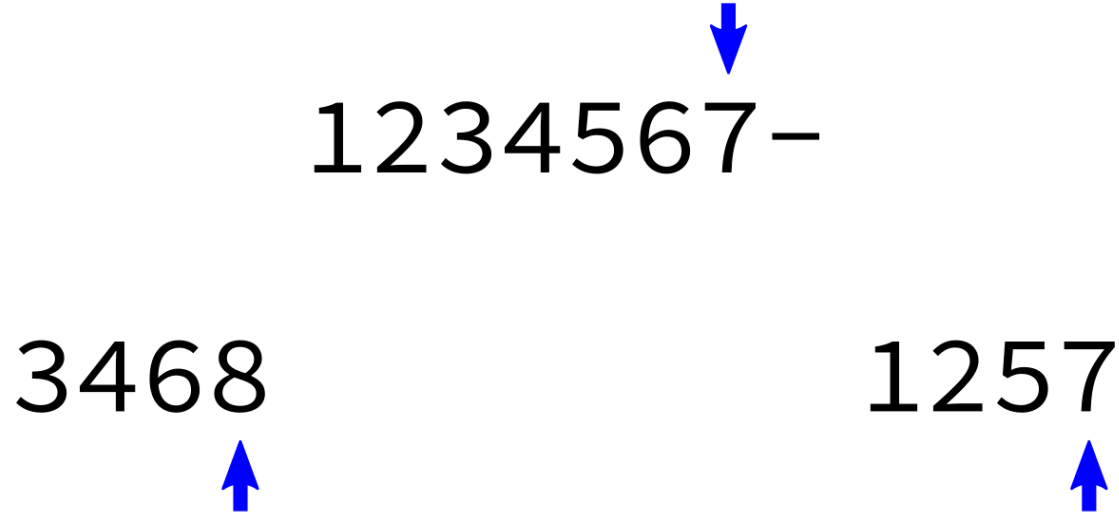

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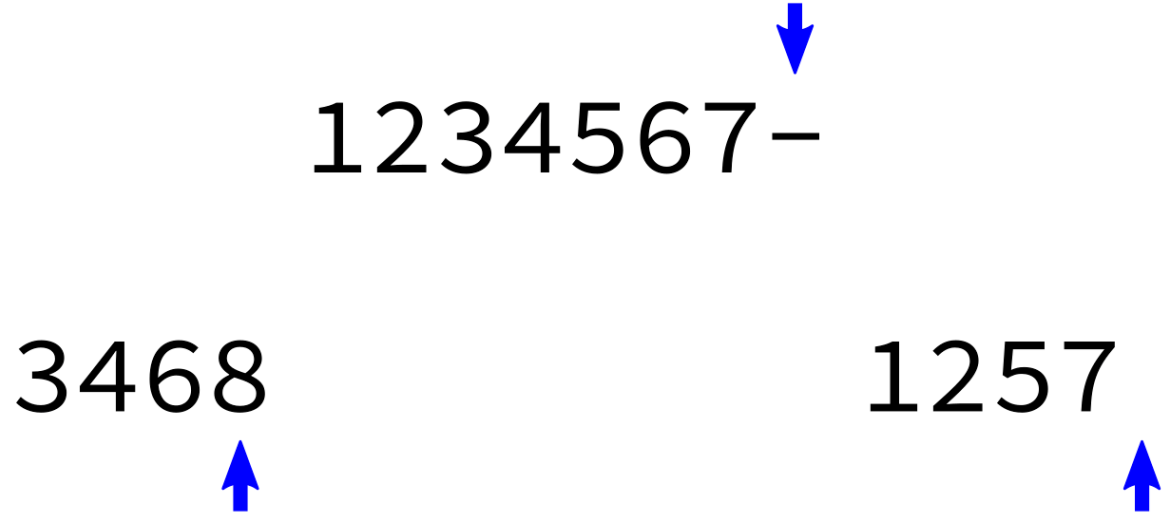

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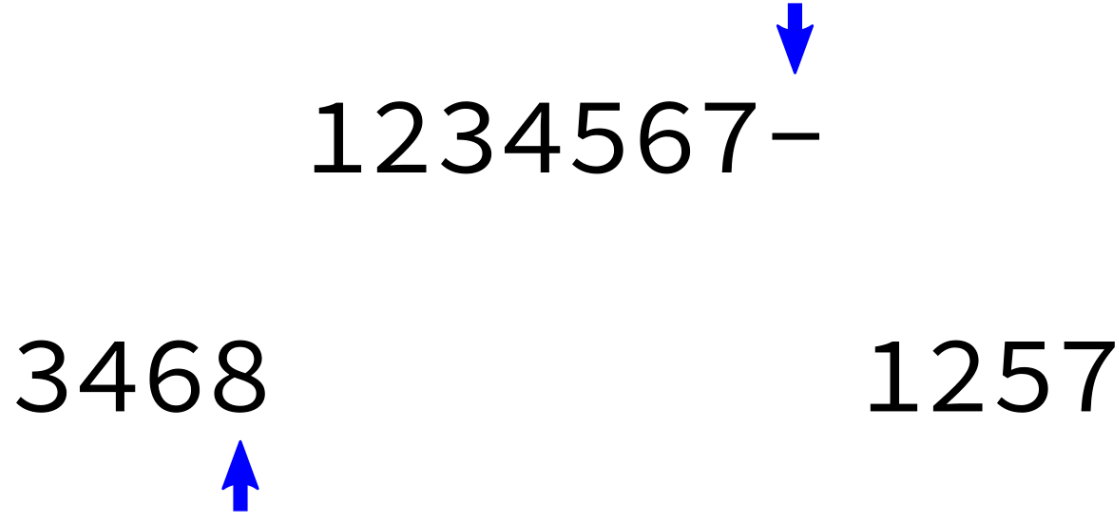

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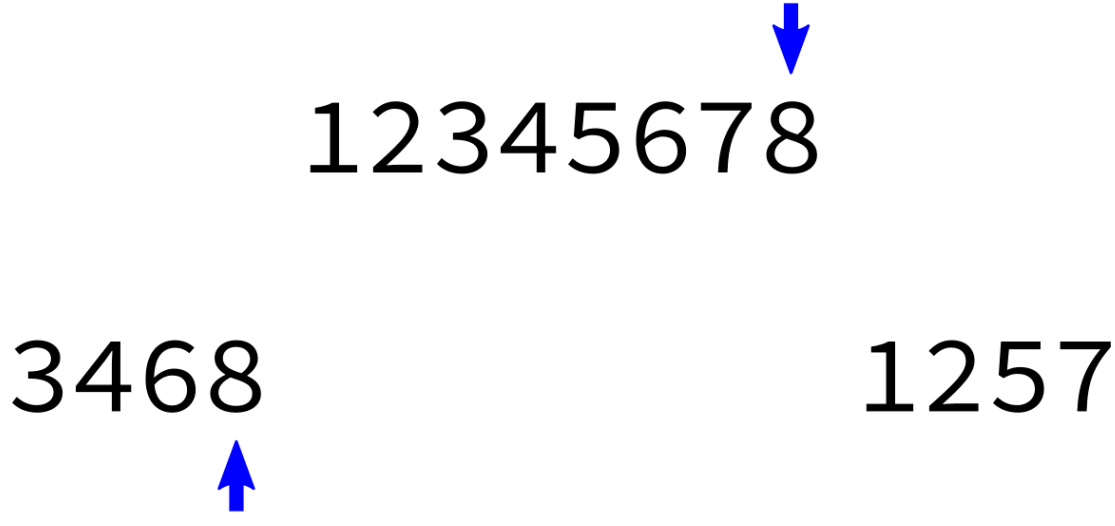

MERGING SORTED LISTS

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MERGING SORTED LISTS

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CODING TIME

Let's implement `mergesort` in Python.

REFERENCES

- Recursion references from Lecture 10.
- Making nice visualizations of sorting algorithms is a cottage industry in CS education. Some you might like to check out:
  - 2D visualization through color sorting by Linus Lee
  - Animated bar graph visualization of many sorting algorithms by Alex Macy
  - Slanted line animated visualizations of mergesort and quicksort by Mike Bostock

REVISION HISTORY

- 2022-02-16 Last year's version of this lecture finalized
- 2023-02-13 Updated for 2023