LECTURE 16

MERGESORT

MCS 275 Spring 2022

Emily Dumas

LECTURE 16: MERGESORT

Course bulletins:

- Project 2 due 6pm central Friday, February 25.
- Worksheet 7 will explore the maze solver / generator in more depth.

PROJECT 2 DISCUSSION

You will write functions (mostly recursive) to enumerate **integer splittings**.

E.g. `1+2+3` and `3+1+2` and `4+2` are splittings of `6`

PLAN

- Discuss the theory of
  - Divide and conquer
  - Sorting
  - Mergesort
- Implement mergesort

DIVIDE AND CONQUER

A strategy that often involves recursion.

- **Split** a problem into parts.
- **Solve** for each part.
- **Merge** the partial solutions into a solution of the original problem.

Not always possible or a good idea. It only works if merging partial solutions is easier than solving the entire problem.

COMPARISON SORT

Suppose you have a list of objects that can be compared with $==, >, <$.

You'd like to reorder them in increasing order.

This problem is called **comparison sort**. There are many solutions.

MERGESORT

A divide-and-conquer solution to comparison sort.

It is a fast solution, often used in practice.

Key: It is pretty easy to take two sorted lists and merge them into a single sorted list.

So, let's divide our list into halves, sort each one (recursively), then merge them.

Now we'll formalize this.

Algorithm `mergesort`:

Input: list `L` whose elements support comparison.

Goal: return a list that contains the items from `L` but in sorted order.

1. If `L` has 0 or 1 elements, return `L`
2. Otherwise, divide `L` into rougly equal pieces `L0` and `L1`.
3. Use recursive calls to sort `L0` and `L1`.
4. Use `merge` to merge these sorted lists and return the result.

MERGESORT EXAMPLE

86435712

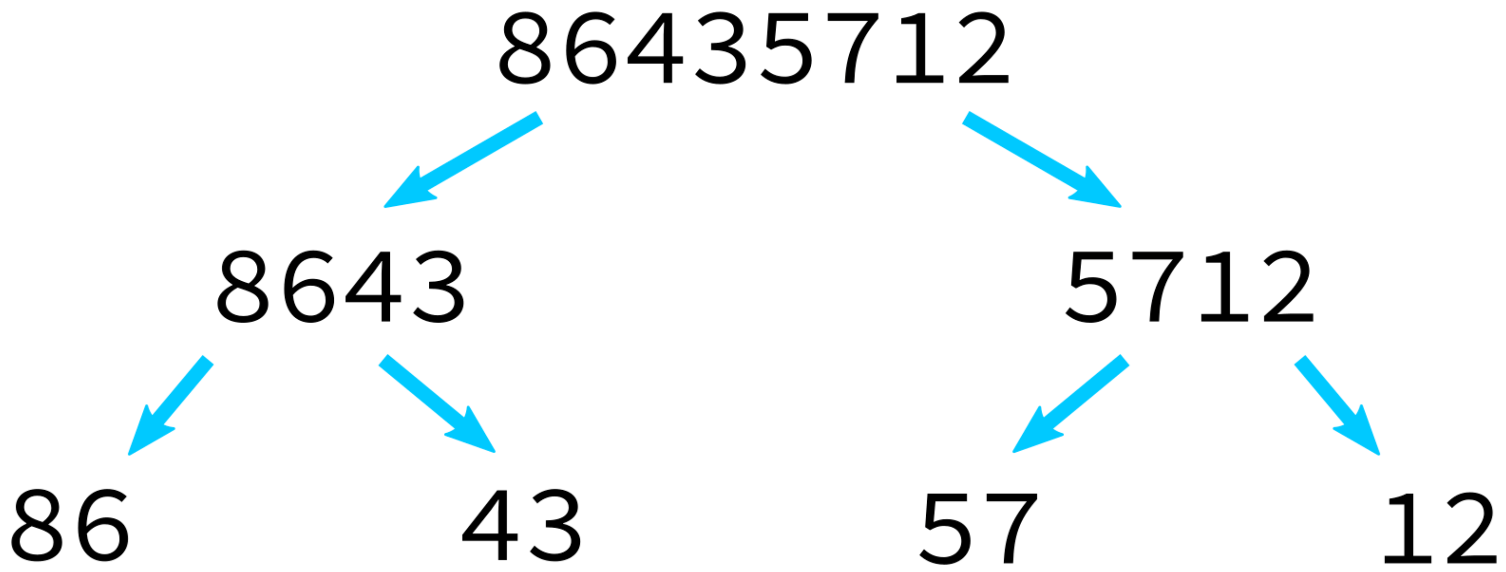
MERGESORT EXAMPLE

86435712

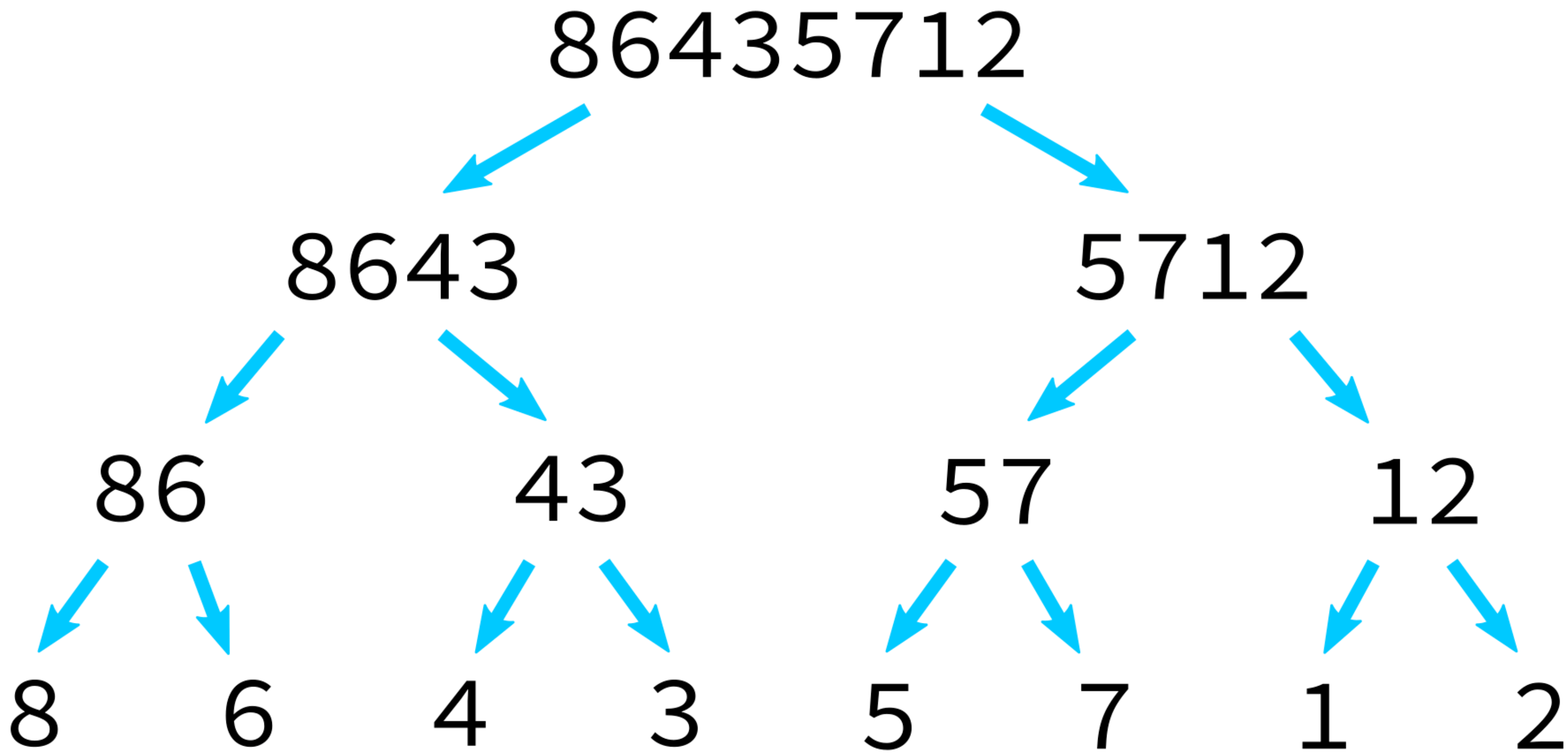
8643

5712

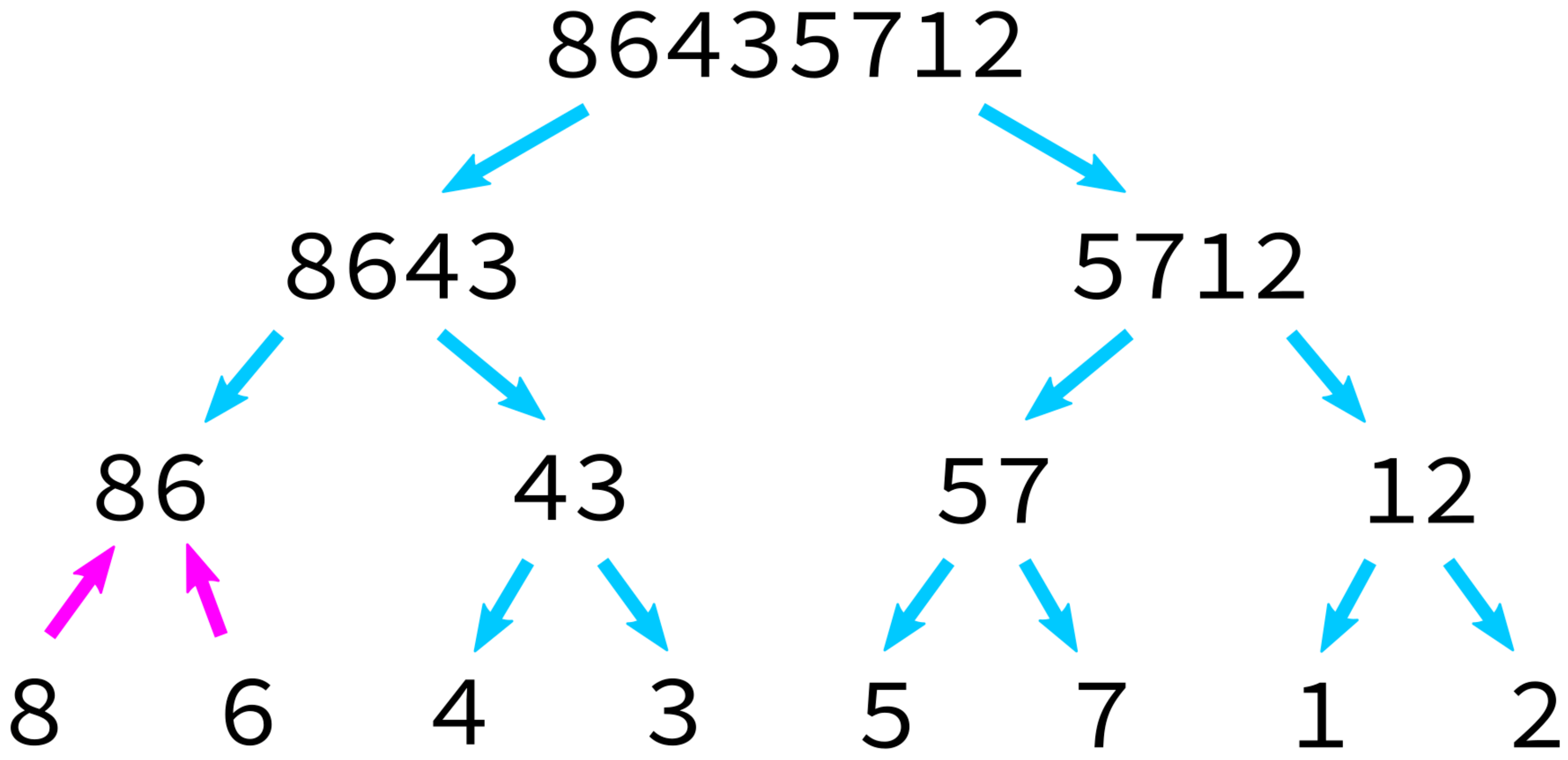
MERGESORT EXAMPLE



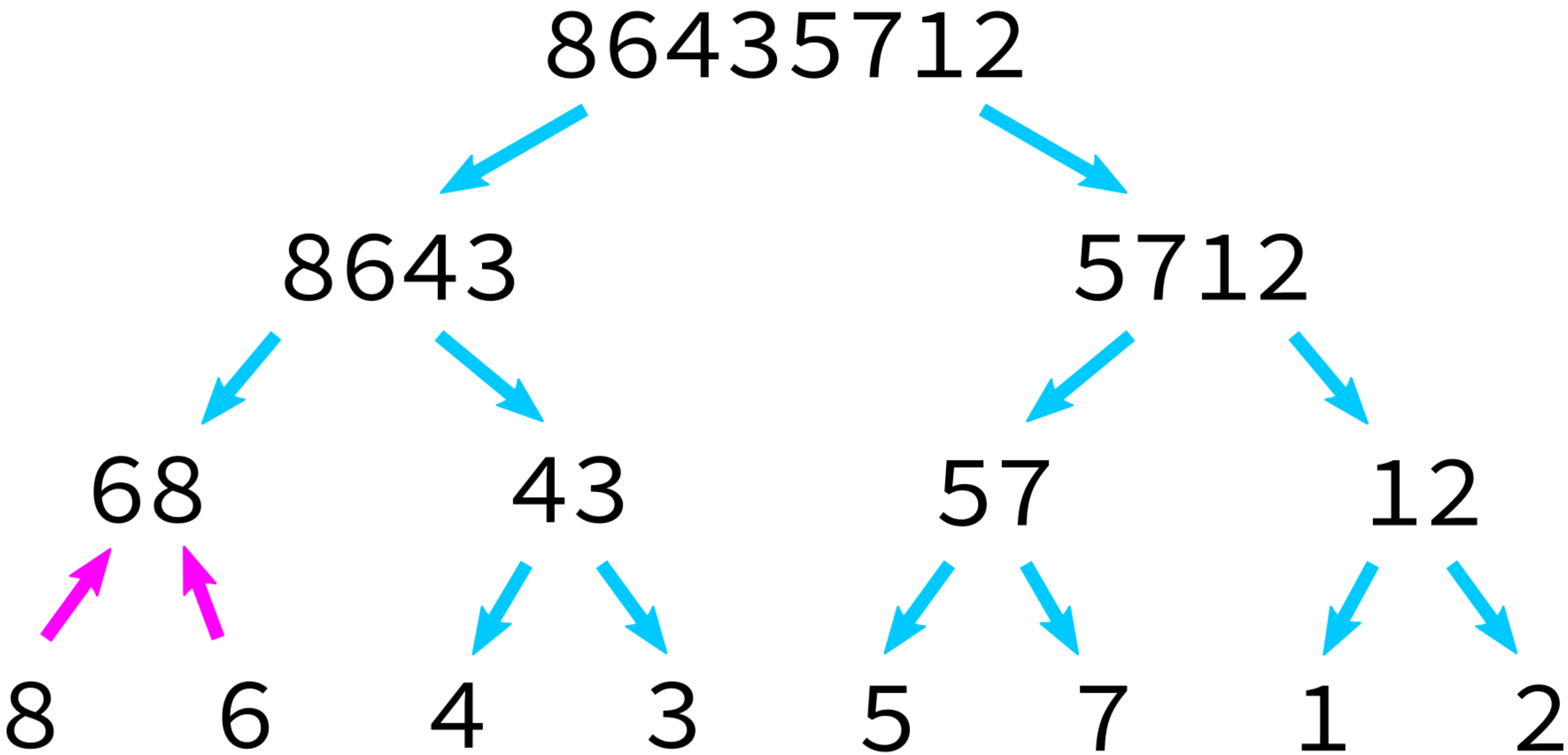
MERGESORT EXAMPLE



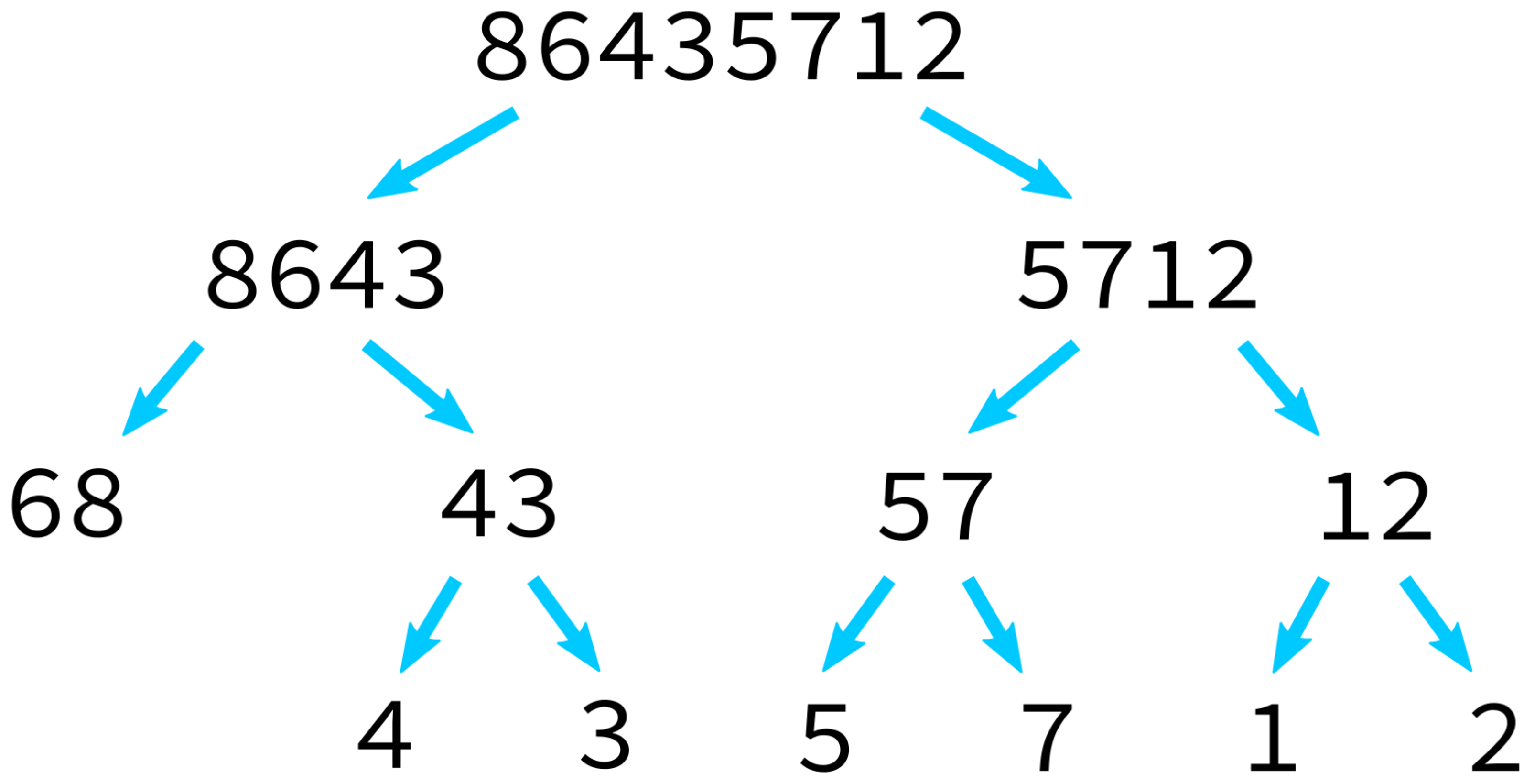
MERGESORT EXAMPLE



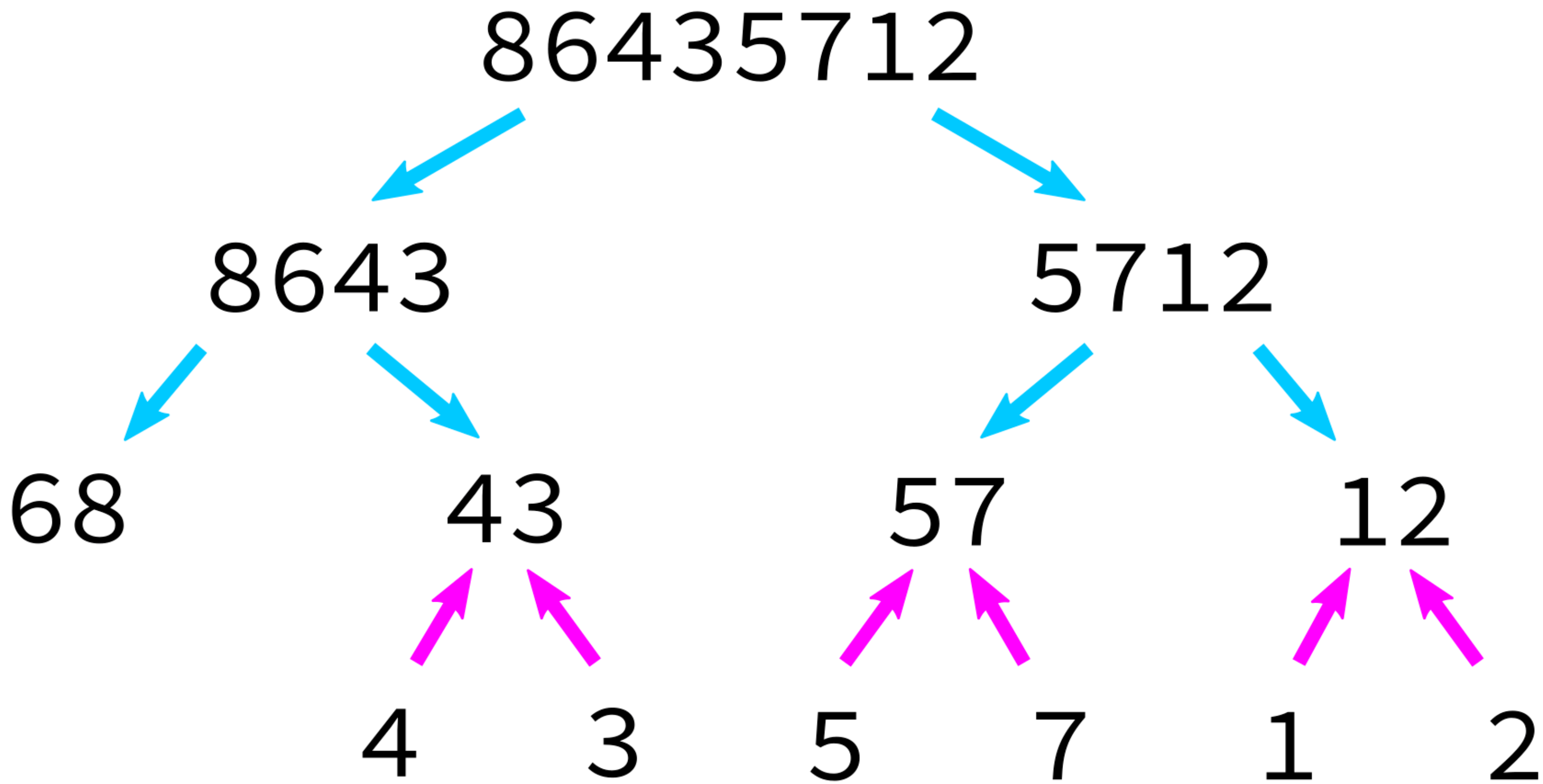
MERGESORT EXAMPLE



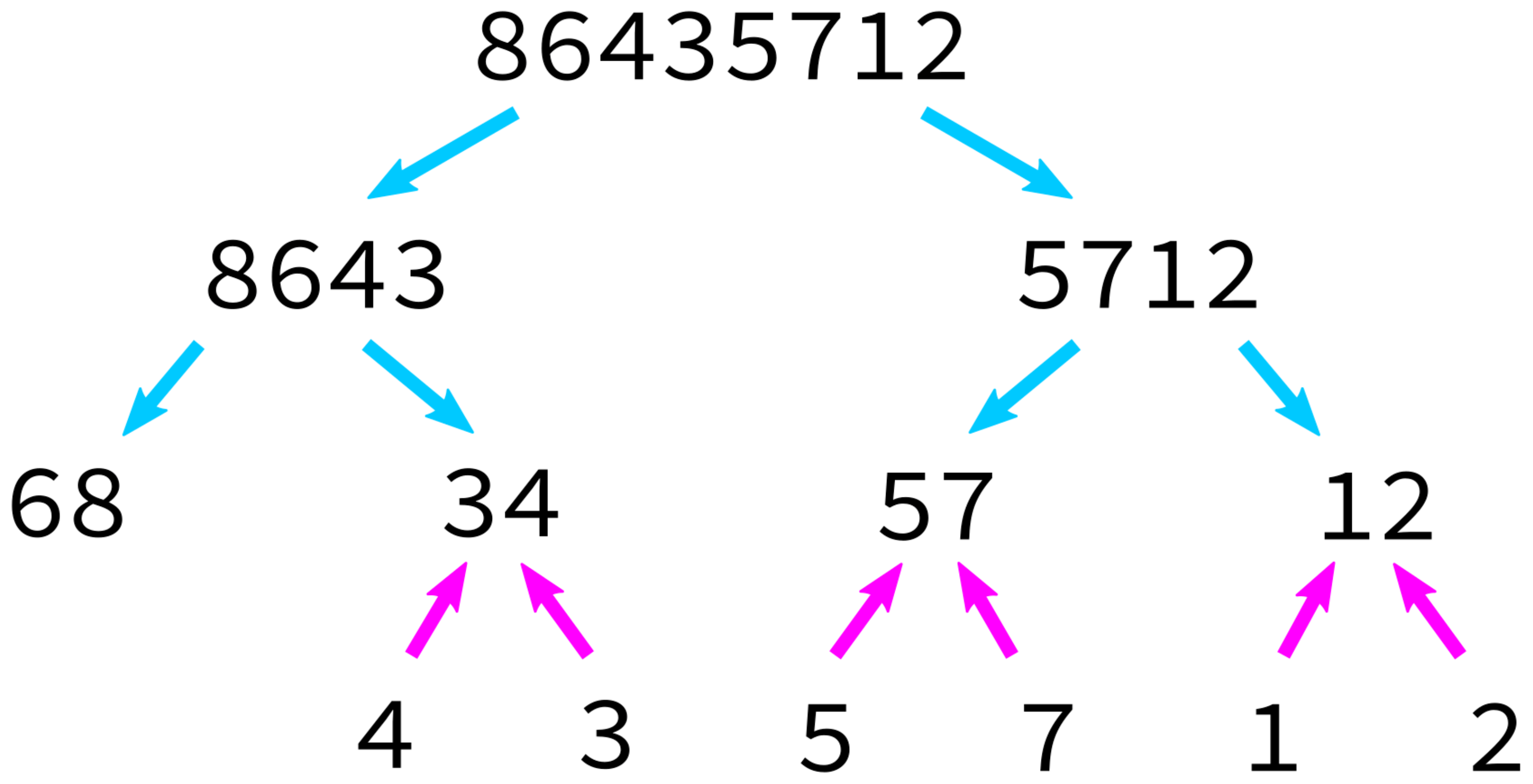
MERGESORT EXAMPLE



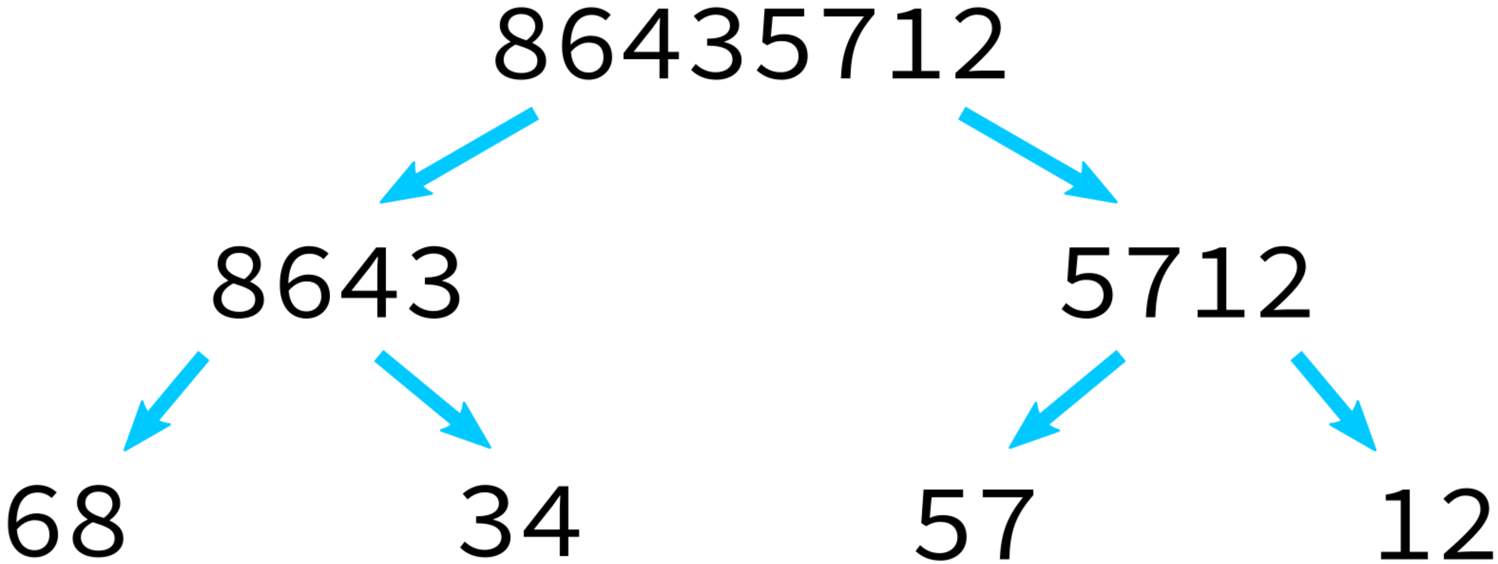
MERGESORT EXAMPLE



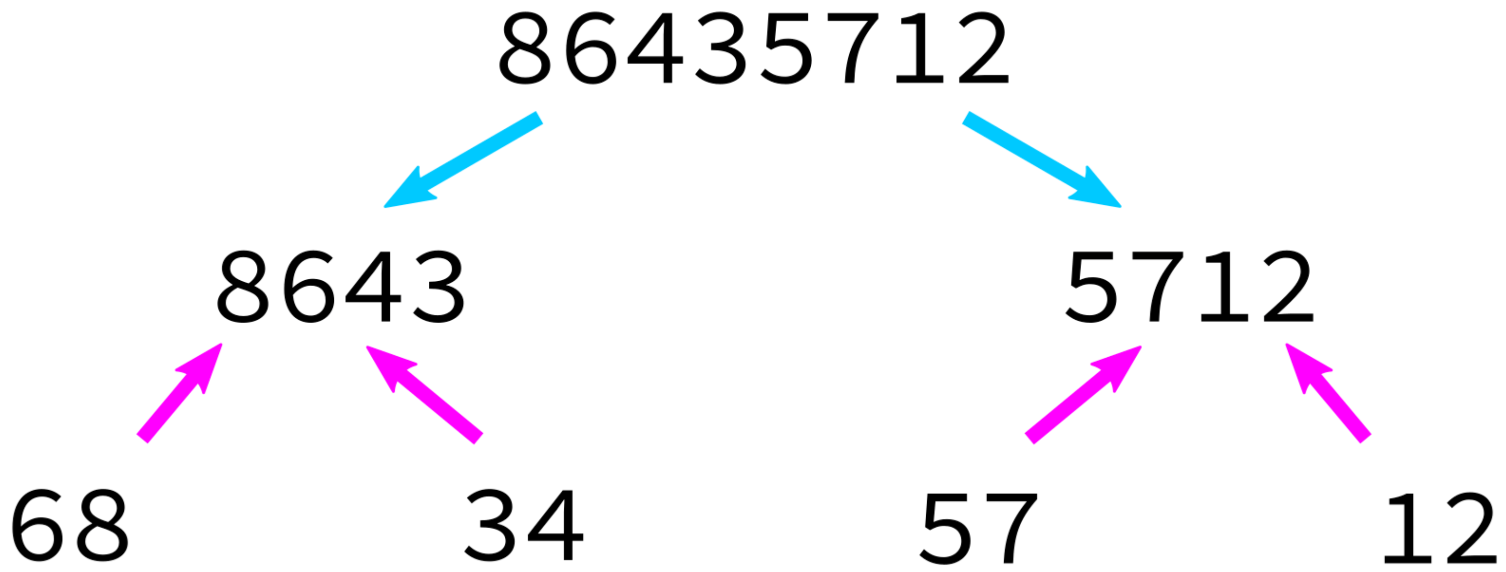
MERGESORT EXAMPLE



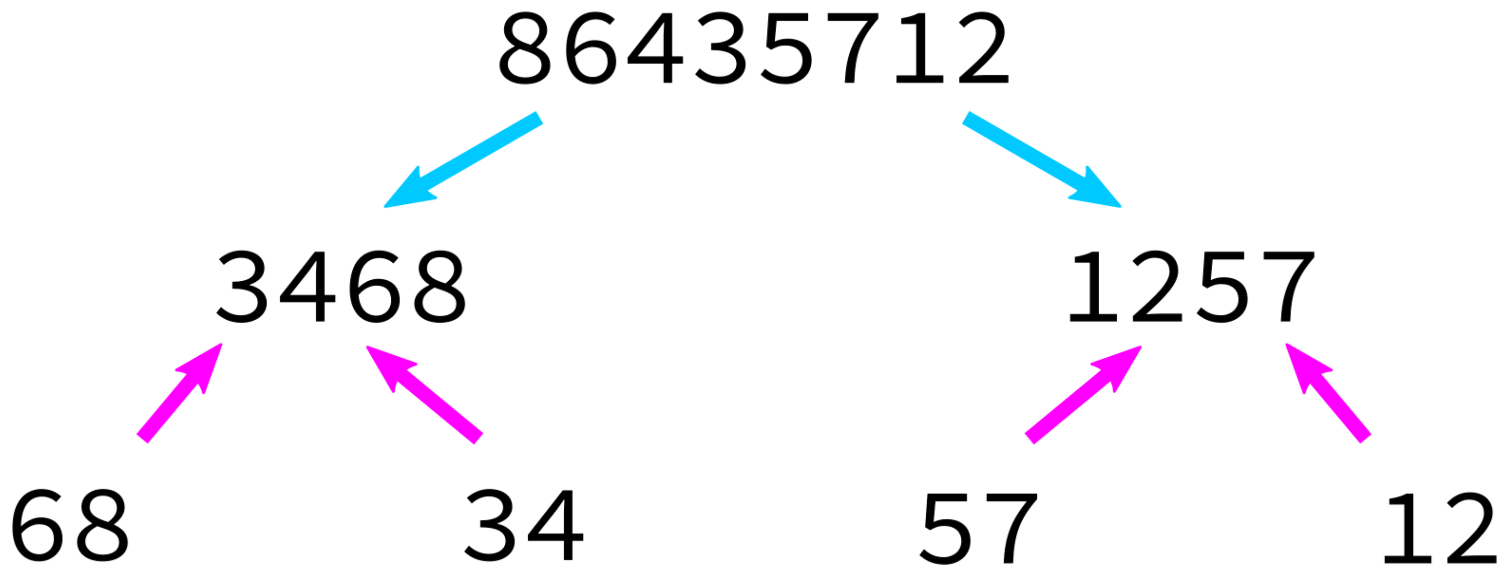
MERGESORT EXAMPLE



MERGESORT EXAMPLE



MERGESORT EXAMPLE



MERGESORT EXAMPLE

86435712

3468 1257

MERGESORT EXAMPLE

12345678

3468 1257

MERGESORT EXAMPLE

12345678

BUT HOW TO MERGE?

This algorithm depends on having a function `merge` that can merge two sorted lists into a single sorted list.

Algorithm `merge`:

Input: sorted lists `L0` and `L1`.

Goal: return a sorted list with same items as `L0+L1`

1. Make a new empty list `L`
2. Make integer variables `i0`, `i1` to keep track of current position in `L0`, `L1` respectively. Set to zero.
3. While `i0 < len(L0)` and `i1 < len(L1)`, do the following:
    - Check which of `L0[i0]` and `L1[i1]` is smaller.
    - Append the smaller one to `L`.
    - Increment whichever one of `i0`, `i1` was used.
4. Append any remaining portion of `L0` to `L`.
5. Append any remaining portion of `L1` to `L`.

MERGING SORTED LISTS

86435712

MERGING SORTED LISTS

86435712

8643

5712

MERGING SORTED LISTS

86435712

3468 1257

MERGING SORTED LISTS

_ _ _ _ _ _ _ _

3468 1257

MERGING SORTED LISTS

```
_ _ _ _ _ _ _ _

3468          1257
```

MERGING SORTED LISTS

```
    1-------

3468        1257
```

MERGING SORTED LISTS

```
        1-------

3468            1257
```

MERGING SORTED LISTS

```
    12------

3468        1257
```

MERGING SORTED LISTS

```
12------

3468        1257
```

MERGING SORTED LISTS

123-----

3468

1257

MERGING SORTED LISTS

```
     123-----

3468          1257
```

MERGING SORTED LISTS

```
1234----

3468        1257
```

MERGING SORTED LISTS

```
1234----

3468        1257
```

MERGING SORTED LISTS

```
12345---

3468        1257
```

MERGING SORTED LISTS

```
        12345---

3468            1257
```

MERGING SORTED LISTS

```
123456--

3468        1257
```

MERGING SORTED LISTS

123456--

3468 1257

MERGING SORTED LISTS

1234567-

3468

1257

MERGING SORTED LISTS

1234567-

3468 1257

MERGING SORTED LISTS

1234567-

3468

1257

MERGING SORTED LISTS

12345678

3468

1257

MERGING SORTED LISTS

12345678

3468 1257

MERGING SORTED LISTS

12345678

CODING TIME

Let's implement `mergesort` in Python.

REFERENCES

- Recursion references from Lecture 13.
- Making nice visualizations of sorting algorithms is a cottage industry in CS education. Some you might like to check out:
    - 2D visualization through color sorting by Linus Lee
    - Animated bar graph visualization of many sorting algorithms by Alex Macy
    - Slanted line animated visualizations of mergesort and quicksort by Mike Bostock

REVISION HISTORY

- 2022-02-16 Initial publication