Math 549: Differentiable Manifolds I – David Dumas – Fall 2022

Homework 14

Due **Monday, December 5**, 2022 at 11:59pm

The same instructions given on homework 1 and 2 apply.

(P1) Let $\mathbb{T}^2 = \{(w,x,y,z) \mid w^2 + x^2 = y^2 + z^2 = 1\}$ with the same orientation described in problem 16-2 from Lee (which was on last week's homework).

(a) Give an explicit example of a 2-form $\alpha \in \Omega^2(\mathbb{R}^4)$ such that $\int_{\mathbb{T}^2} \alpha = 1$.

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be any smooth function. Show that $\beta = f(w,y)\, dw \wedge dy$ satisfies $\int_{\mathbb{T}^2} \beta = 0$.

(P2) (The following is a slightly modified version of Problem 17-5 in Lee's book, on page 465. The change just makes part of the problem optional.)

For each $n \geq 1$ and $p \geq 0$ compute the dimension of $H^p(\mathbb{R}^n \setminus \{e_1, -e_1\})$.

Optionally, for extra credit: In each case where this vector space is nontrivial, give closed differential forms whose cohomology classes form a basis.

(P3) Let C and D be circles in $\mathbb{R}^2$ whose radii sum to less than the distance between their centers. As in the midterm exam, consider the map $F : C \times D \to \mathbb{S}^1$ that takes (c,d) to the unit vector in the direction of $d - c$. Thus we obtain a linear map $F^* : H^1(S^1) \to H^1(C \times D)$. Since $H^1(\mathbb{S}^1) \simeq \mathbb{R}$, this linear map is either the zero map or else it is injective. Which is it?

Revision history:

- 2022-12-02 Make the basis part of 17-5 optional.