Math 549: Differentiable Manifolds I – David Dumas – Fall 2022

Homework 13

Due **Tuesday, November 29**, 2022 at 11:59pm (note usual due date!)

The same instructions given on homework 1 and 2 apply.

(P1) (a) Suppose that ω is an orientation form on a compact manifold M of dimension n. Show that there **does not exist** an $(n-1)$-form η such that $\omega = d\eta$.

(b) Find $\eta \in \Omega^{n-1}(\mathbb{R}^n)$ so that $d\eta = dx_1 \wedge \cdots \wedge dx_n$.

(P2) Let $M = \{(x,y,z) \,|\, x^2+y^2+z^2 = 1,\ z \leq 0\}$ with the orientation arising from the standard orientation of $\mathbb{R}^3$ and the normal vector field $N(x,y,z) = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$. Let $\alpha = dx \wedge dy + dy \wedge dz$.

(a) Compute $\int_M \alpha$ directly (i.e. without using Stokes' theorem)

(b) Compute $\int_M \alpha$ using Stokes' theorem