Homework 12

Due Monday, November 21, 2022 at 11:59pm

The same instructions given on homework 1 and 2 apply.

(P1) Show that each condition below implies an n-dimensional manifold M is orientable.
- (a) M has a Lie group structure.
- (b) $M = U \cup V$ where U and V are orientable open submanifolds of M and $U \cap V$ is connected.

(P2) Suppose N is a smooth manifold. Recall that a local coordinate system $(x_1, \ldots, x_n)$ on N (defined on a set U) gives a local coordinate system $(x_1, \ldots, x_n, c_1, \ldots, c_n)$ on TN (defined on TU) in which $(x_1, \ldots, x_n, c_1, \ldots, c_n)$ represents the point $(p, v) \in TU$ where p has coordinates $(x_1, \ldots, x_n)$ and $v = \sum_i c_i \left.\frac{\partial}{\partial x_i}\right|_p$.
- (a) Show that the transition functions of the atlas for TN obtained this way are orientation-preserving (as maps between open sets in $\mathbb{R}^{2n}$).
- (b) Conclude that TN is orientable.

(P3) Problem 16-2 from Lee, on page 434. (Integrate $xyz\,dw \wedge dy$ over the Clifford torus.)

Revision history:
- 2022-11-21 Fix header.