Math 549: Differentiable Manifolds I – David Dumas – Fall 2022

Homework 1

Due Monday, August 29, 2022 at 11:59pm[1]

Instructions:
   (i) Write solutions with complete sentences that explain your answer. Don't just write a series of formulas and symbols.
  (ii) Write as if the audience is another student in the class.
 (iii) If a problem asks for a proof, begin it with"Proof:" and end with □ or QED.
  (iv) Submit your solutions to Gradescope. (Do not give paper to course staff.)
   (v) Do not include this document as part of what you submit.
  (vi) Label solutions with the problem numbers from the list below, e.g. "(P3)". If a problem comes from the textbook, you can also include its number from the book if you want, but this is optional whereas the P-number is required.

Notes:
   • I will include full instructions on the first two assignments, after which they will become the "standing instructions" that apply all the time (unless an assignment indicates otherwise).

(P1) Read Example 1.31 from Chapter 1 in Lee. Then, complete the related **problem 1-7 in Chapter 1** (on page 30).

(P2) Read Example 1.5 and Example 1.33 from Chapter 1 in Lee, which concern the real projective space. Then, complete the related **problem 1-9 in Chapter 1** (on page 30).

(P3) Read Example 1.23 in Lee's book, which describes a smooth atlas on $\mathbb{R}$ different from the "usual" one. Then complete the related **problem 2-5 in Chapter 2** (on page 48) which asks questions about smooth functions for this structure.

(P4) **Problem 2-6 in Chapter 2** (on page 48). (Like one of the earlier exercises on this assignment, this one relies on a careful study of Example 1.33.)

(P5) Construct an explicit smooth atlas for the subspace of $\mathbb{R}^3$ given by the equation $x^2 + y^2 - z^2 = 1$, verifying that the charts you specify are smoothly compatible.

---

[1]Please don't lose sleep in order to keep working right up to the late night deadline; I made it 11:59pm to give you flexibility but hope you will stick to the working hours that are best for you.