Math 549: Differentiable Manifolds I – David Dumas – Fall 2022

Final Exam

Wednesday, December 7

(P1) Consider the vector field $V = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} - 2z\frac{\partial}{\partial z}$ on $\mathbb{R}^3$.

(a) Compute Vf where $f(x,y,z) = x + y^2$

(b) Compute the Lie bracket $[V, \frac{\partial}{\partial z}]$.

(c) Compute the flow of V.

(P2) Consider these differential forms on $\mathbb{R}^3$:

$$\alpha = dy \wedge dz + z\,dx \wedge dz$$
$$\beta = x\,dy - y\,dx$$

Also, let $\gamma : \mathbb{R} \to \mathbb{R}^3$ be the map $F(t) = (t, t^2, 0)$.

(a) Compute $\alpha \wedge \beta$.

(b) Compute $d\alpha$.

(c) Compute $d\beta$.

(d) Compute $\gamma^*(\alpha)$.

(e) Compute $\gamma^*(\beta)$.

(P3) Suppose M and X are smooth manifolds and that there are three smooth maps

$$A : M \to X,$$
$$B : X \to X,$$
$$C : X \to M$$

such that $F = C \circ B \circ A : M \to M$ is a diffeomorphism. Does this imply that any of the maps A, B, C is an immersion? And does it imply that any of A, B, C is a submersion? Justify your answers.

In addition to any arguments, examples, or justifications you write, please include a table like this one that summarizes your answer

	A	B	C
Must be an immersion?			
Must be a submersion?			

writing "yes" or "no" in each box as appropriate.