LECTURE 19

TREES

MCS 275 Spring 2021

Emily Dumas

LECTURE 19: TREES

Course bulletins:

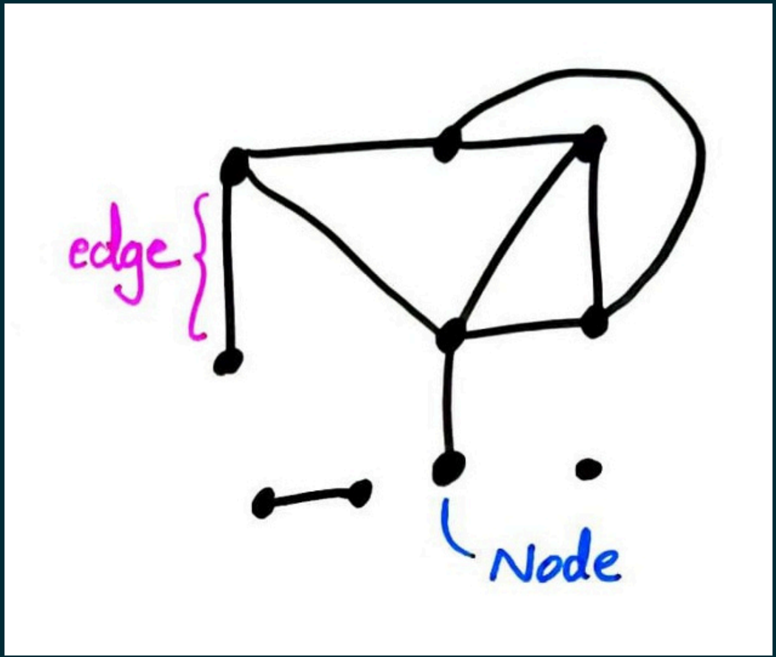
- Project 2 due 6pm CST Friday. Autograder open.
- Lecture 18 video has fixes not seen in live lecture (which I'll also tell you about today).

PLAN

- Finish up a bit of material intended for Lecture 18.
- Discuss trees: in general, in CS, in Python

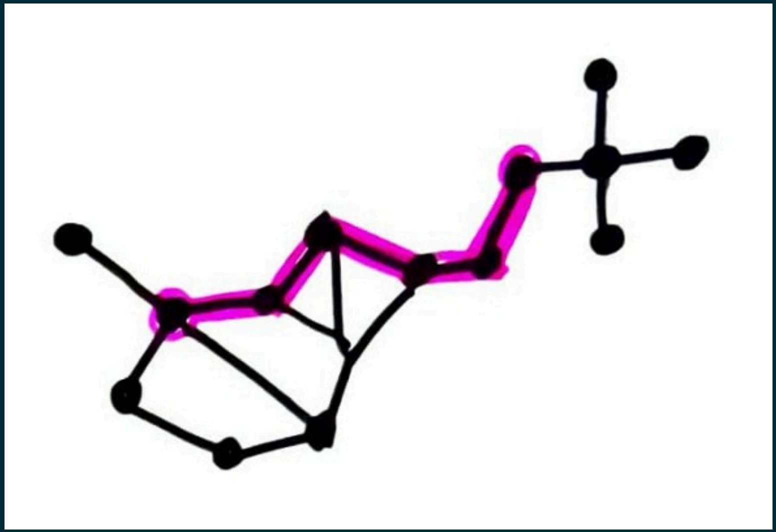
GRAPHS

In mathematics, a graph is a collection of **nodes** (or vertices) and **edges** (which join pairs of nodes).



CONNECTIVITY

A graph is **connected** if every pair of nodes can be joined by *at least* one path.



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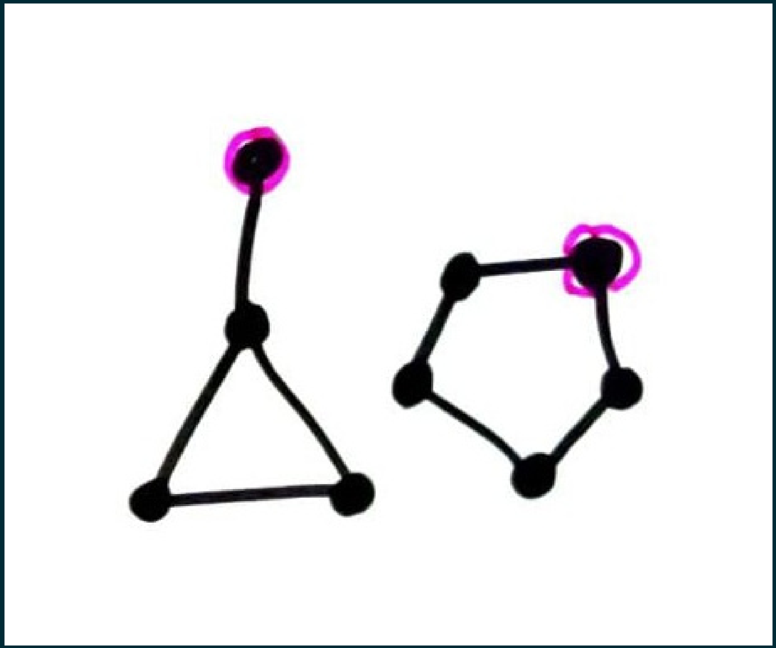

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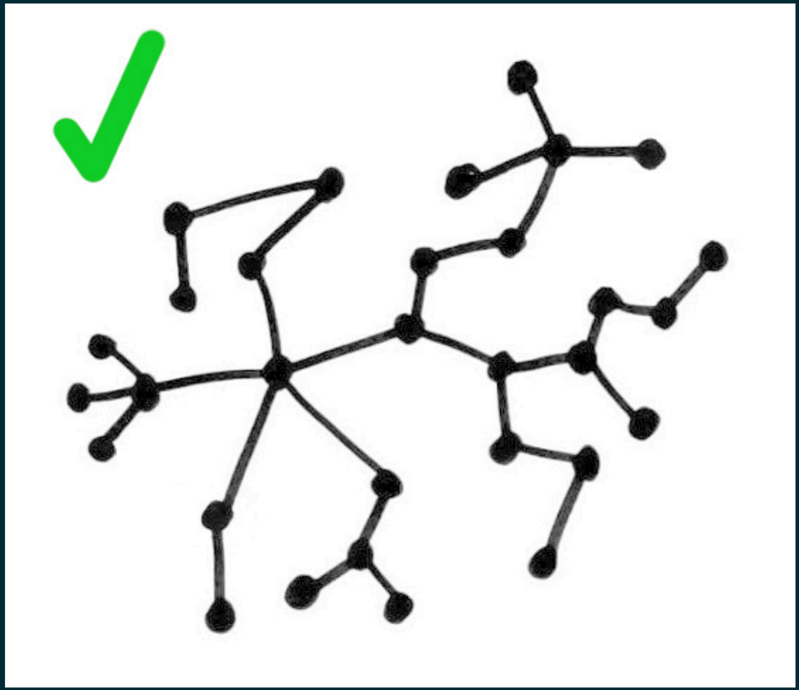

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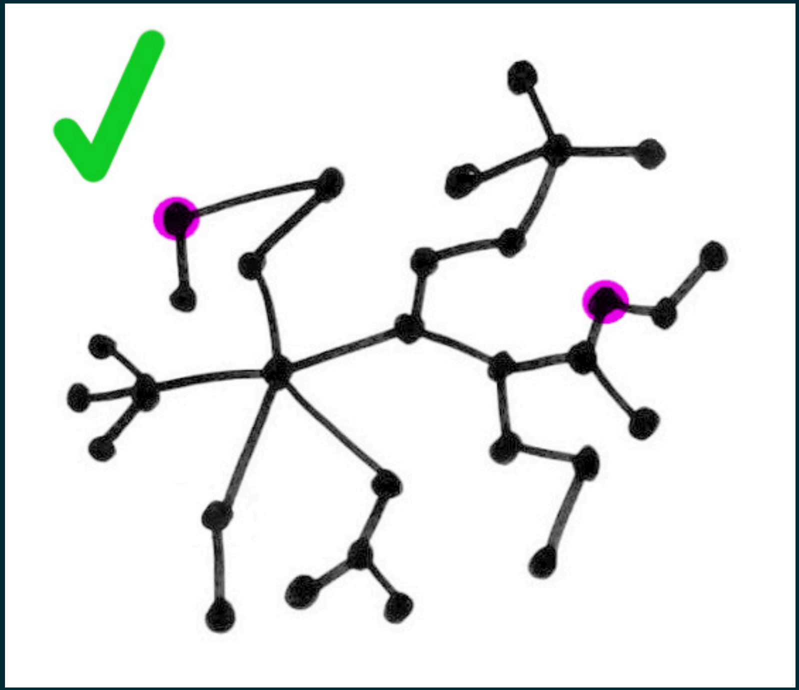

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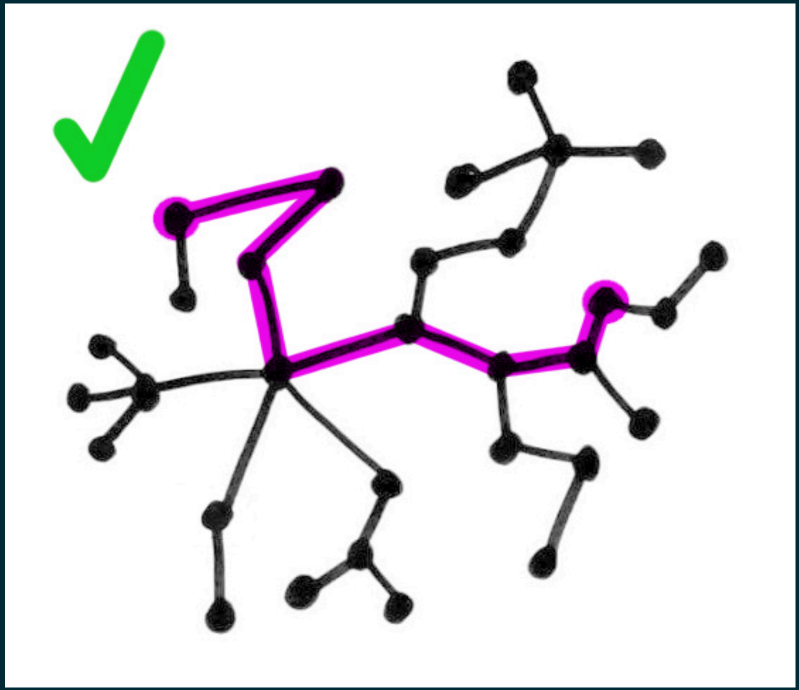

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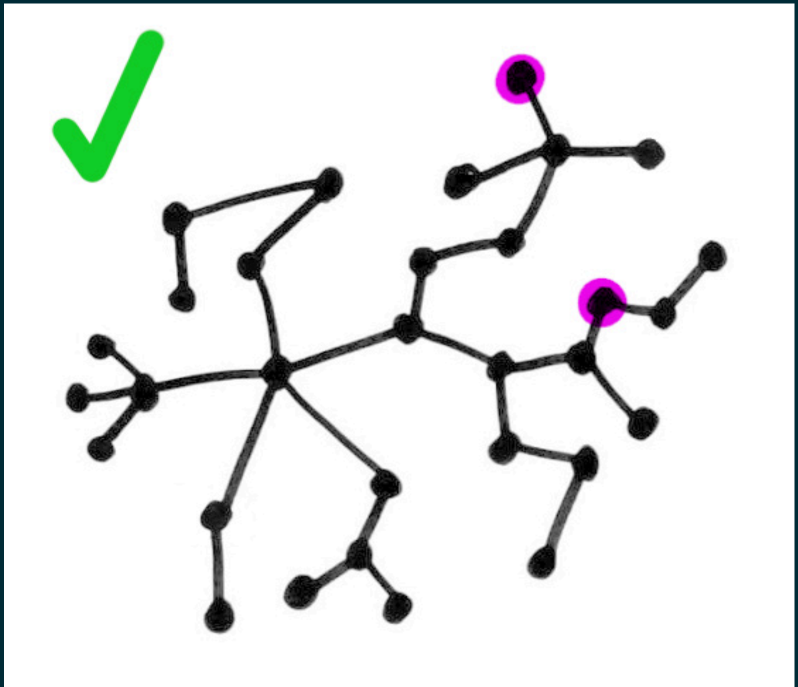

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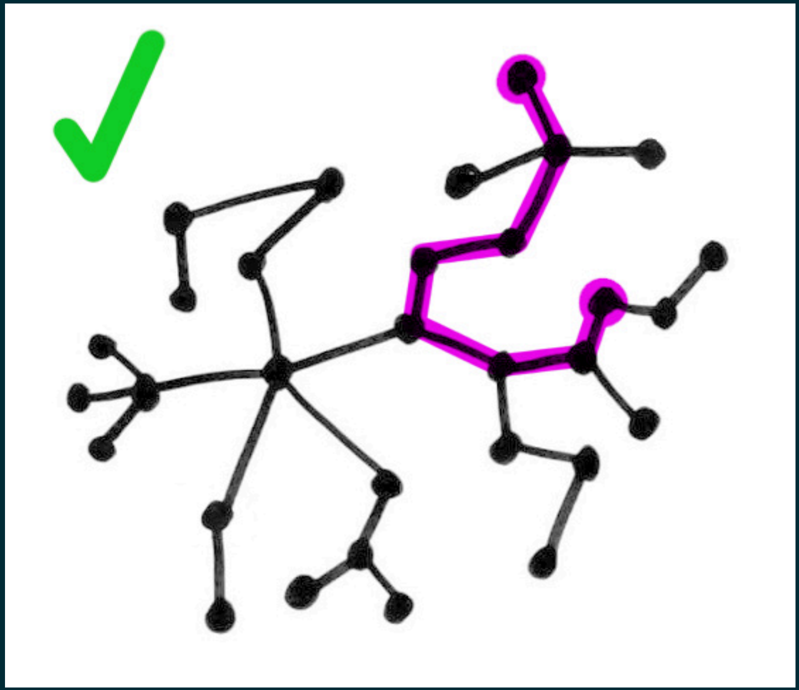

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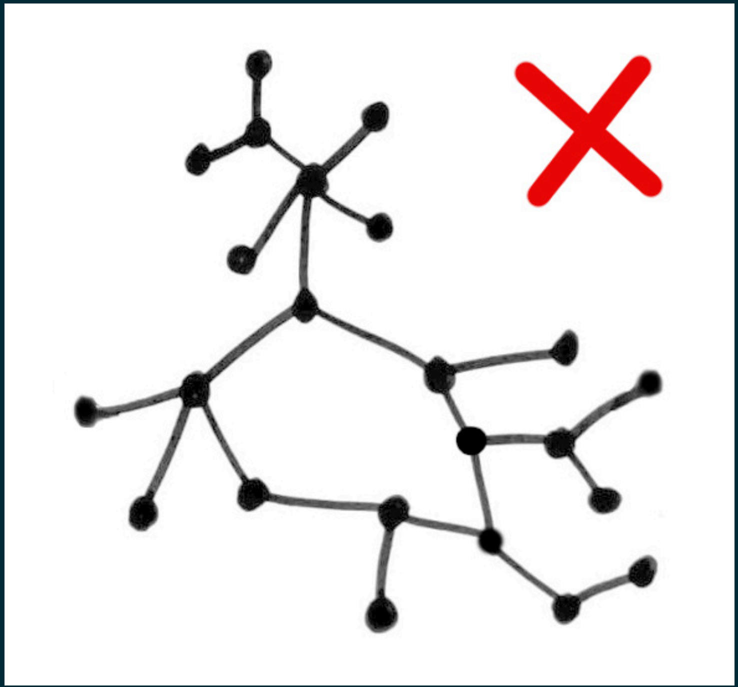

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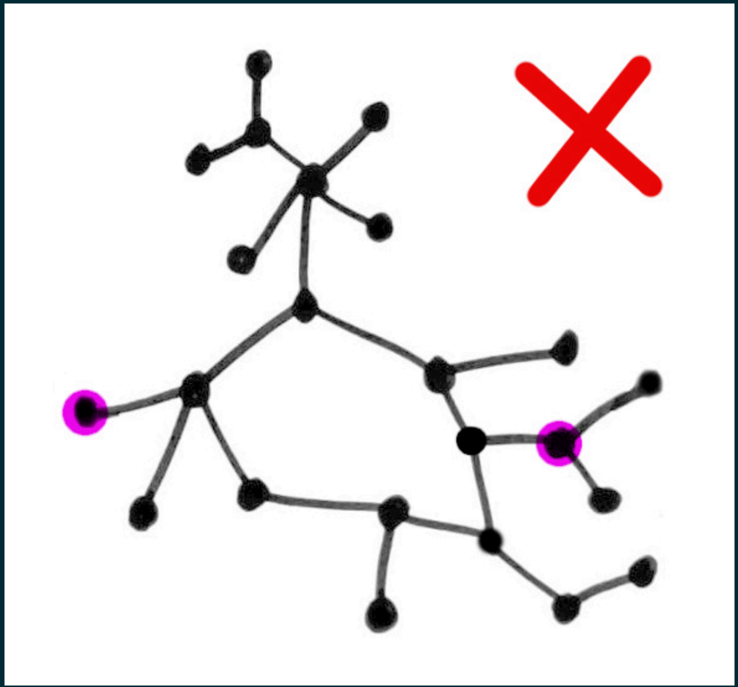

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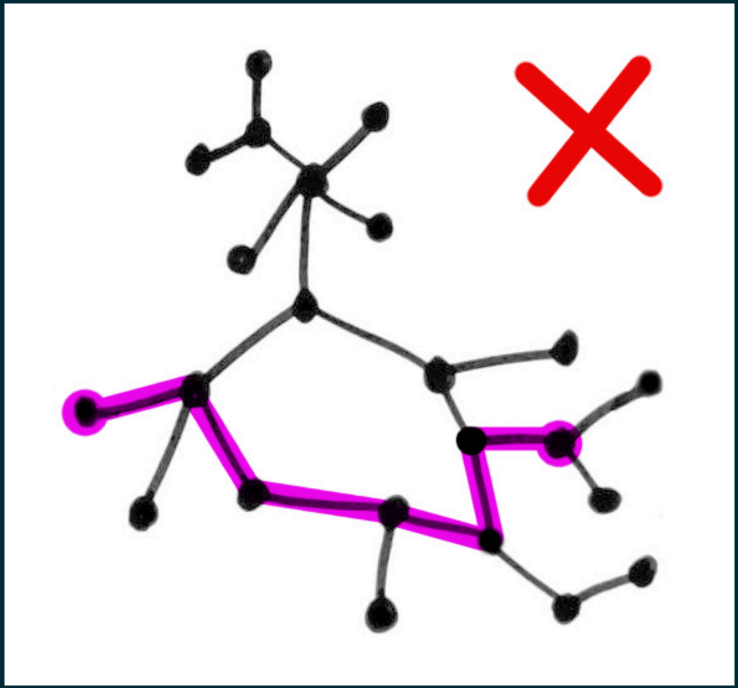

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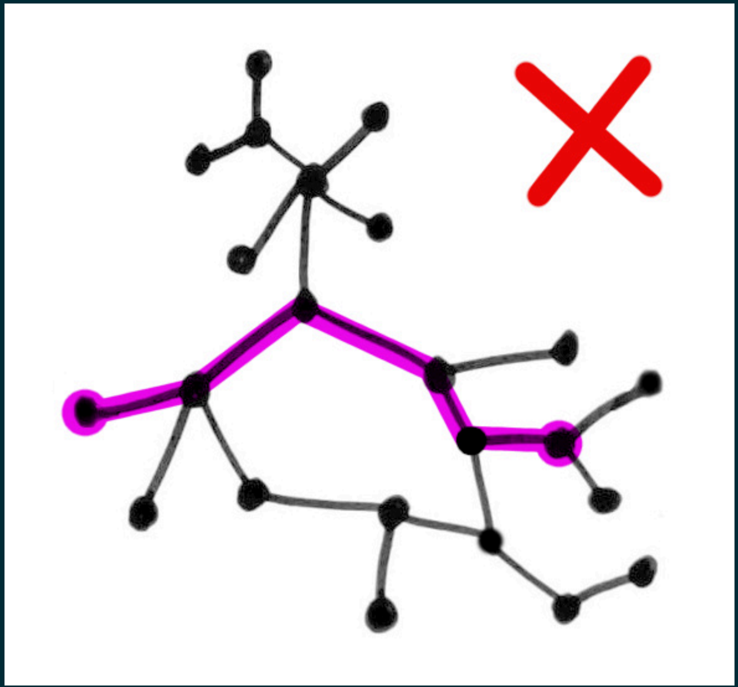
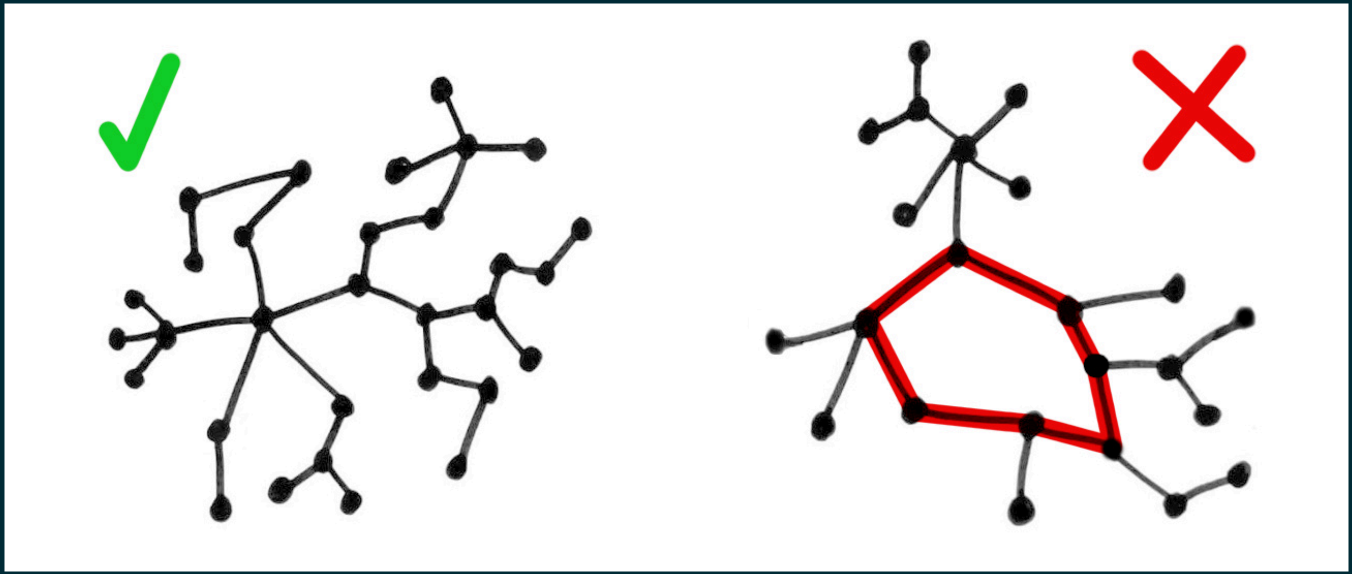


Equivalently, a tree is a **connected graph** with **no loops**.

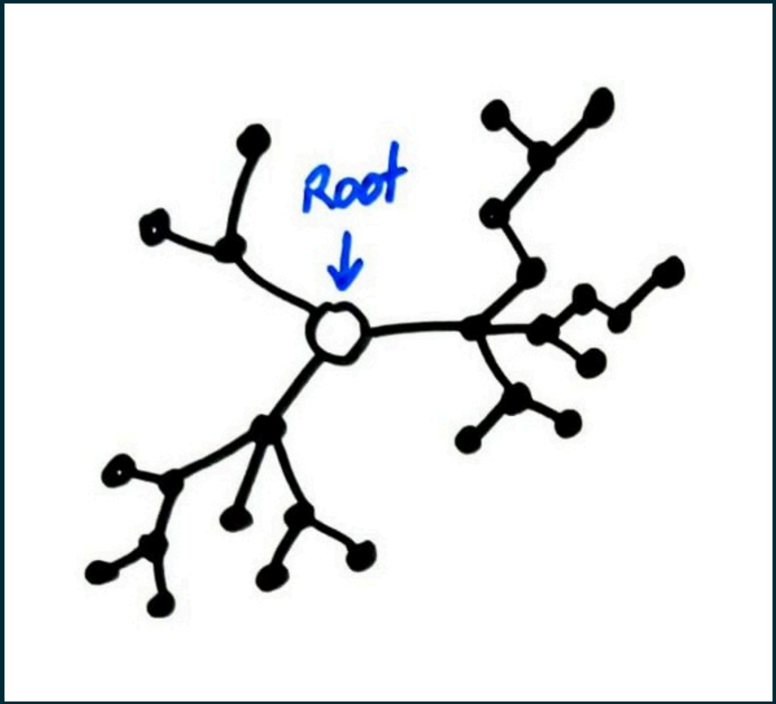


Equivalently, a tree is a connected graph that becomes disconnected if any edge is removed.

(Exercise: Prove this is an equivalent definition!)

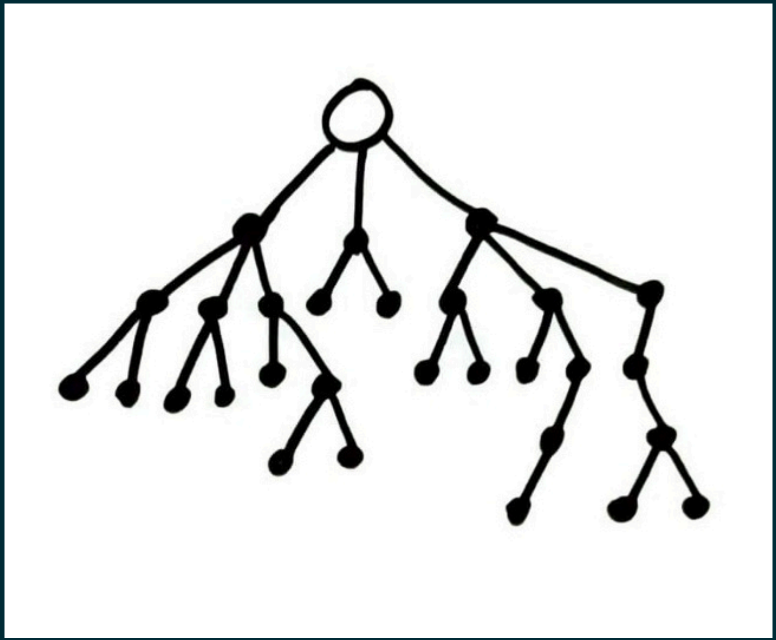
ROOTS AND DIRECTIONS

The trees considered in CS usually have one node distinguished, called the **root**.



There's nothing special about the root except that it is labeled as such. Any node of a tree could be chosen to be its root node.

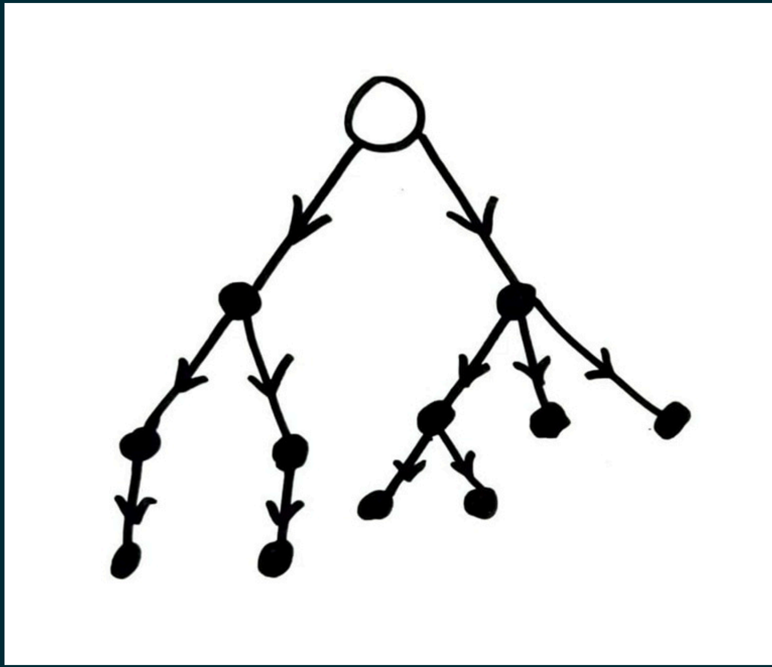
Such *rooted trees* are usually drawn with the root at top



and vertices farther from the root successively lower.

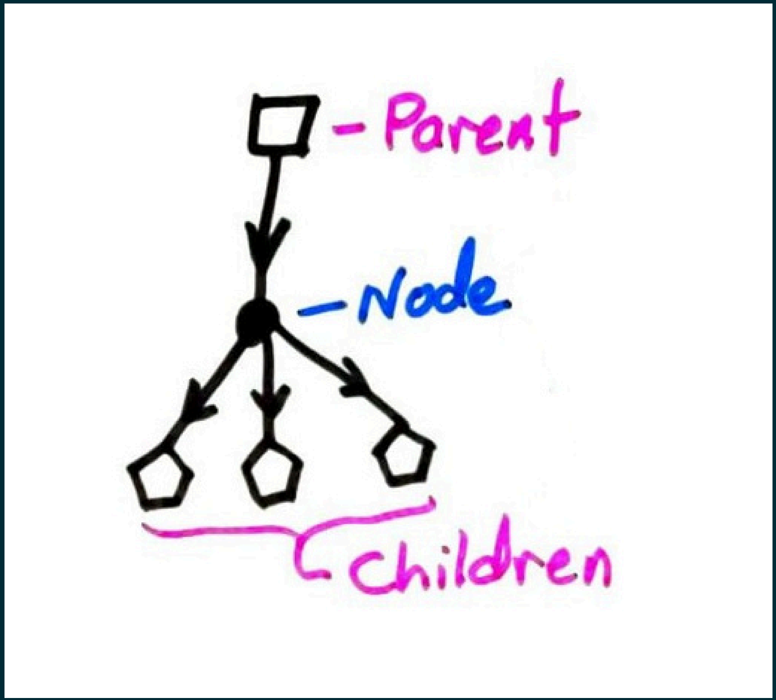
This convention is probably inspired by the way trees look in the natural world.

Choosing a root lets us orient all of the edges so they point away from it.



Hence the usual way of drawing a tree will have these arrows pointing downward.

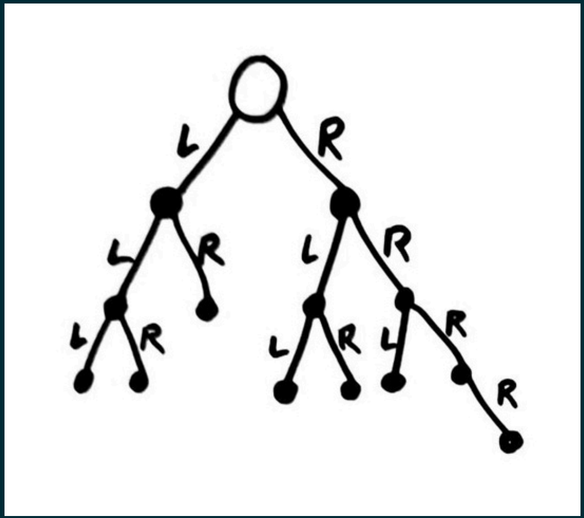
Each node (except the root) has an incoming edge, from its **parent** (closer to the root).



Each node may have one or more outgoing edges, to its **children** (farther from the root).

BINARY TREES

In CS, a binary tree is a (rooted) tree in which every node has ≤ 2 children, labeled "left" and "right".



Horizontal relative position is used to indicate this labeling, rather than explicitly writing it on the edges.

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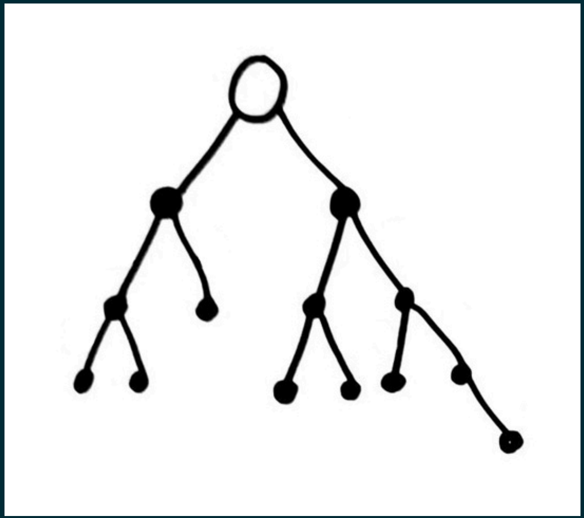

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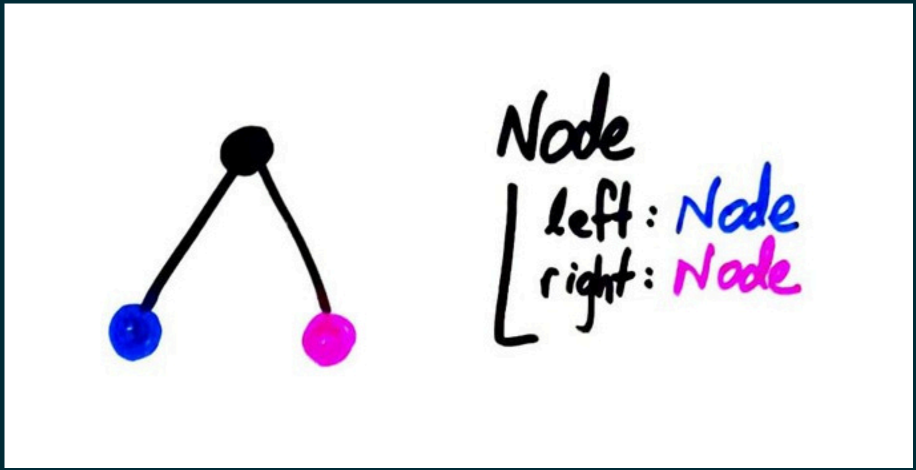
REPRESENTATION

How can we store a tree in Python?



Make a class `Node`, with attribues `left` and `right` that can be `None` or other `Node` objects.

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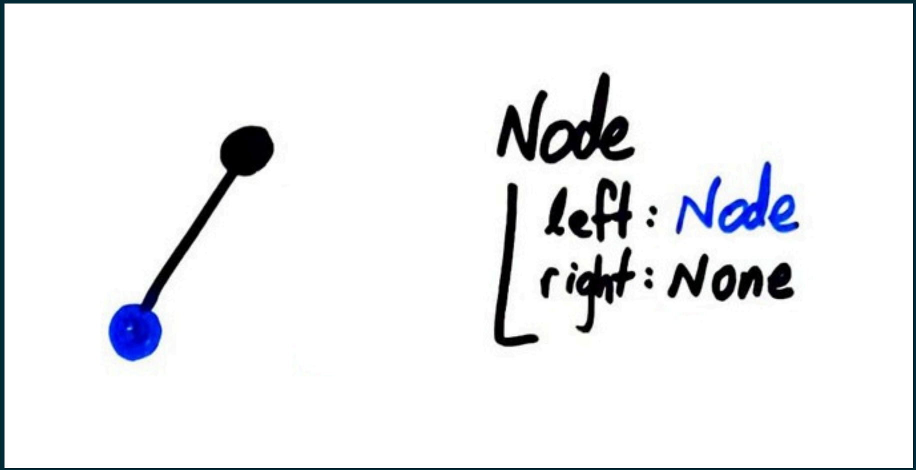

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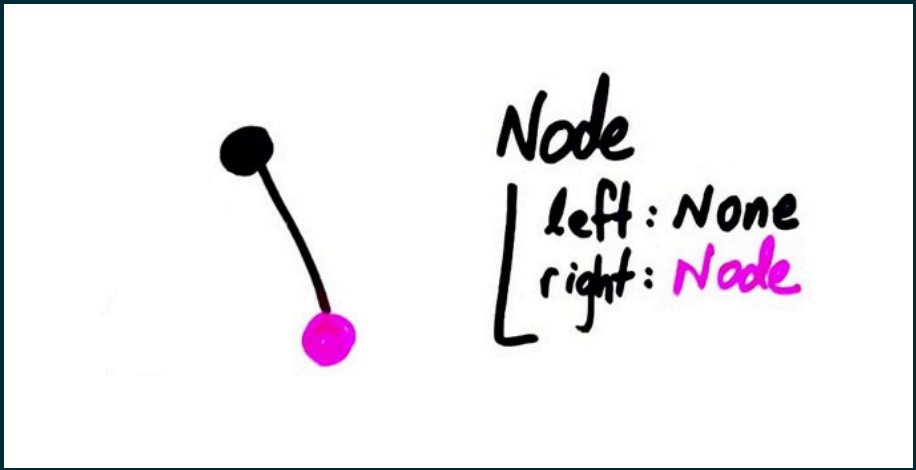

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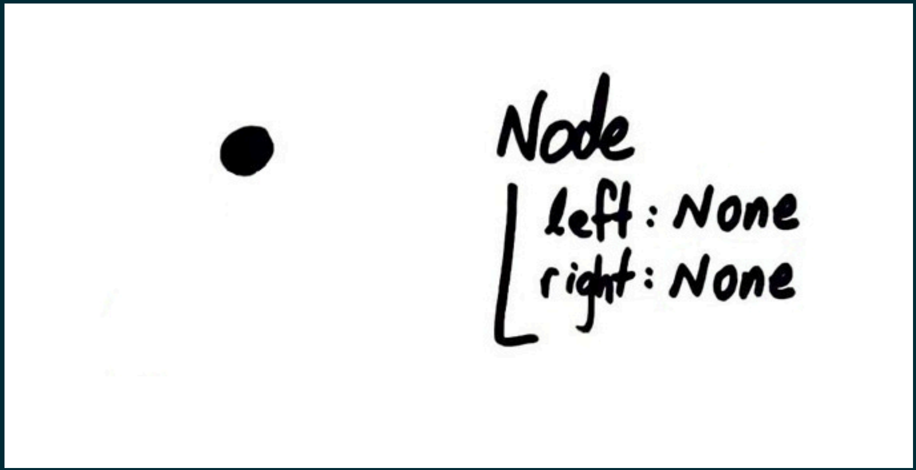

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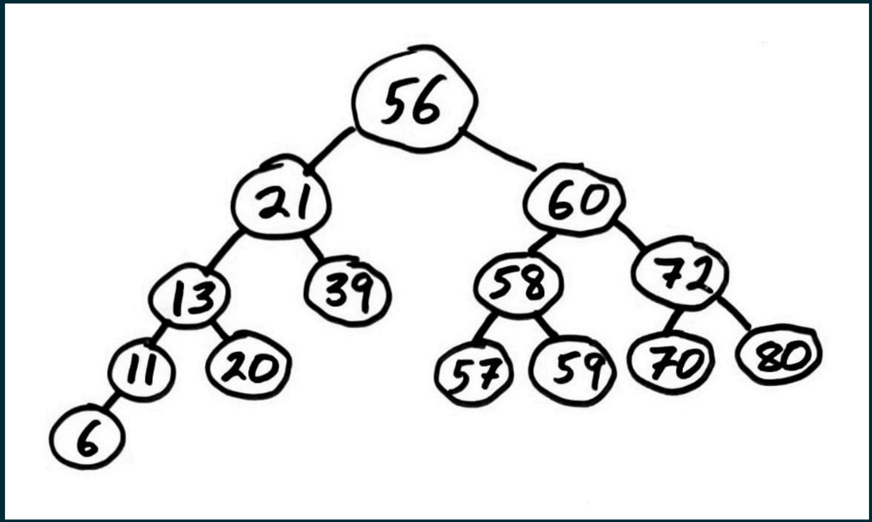
WHY?

We can also store additional information in the nodes of a binary tree. If present, this is called the **key** or *value* or *cargo* of a node.

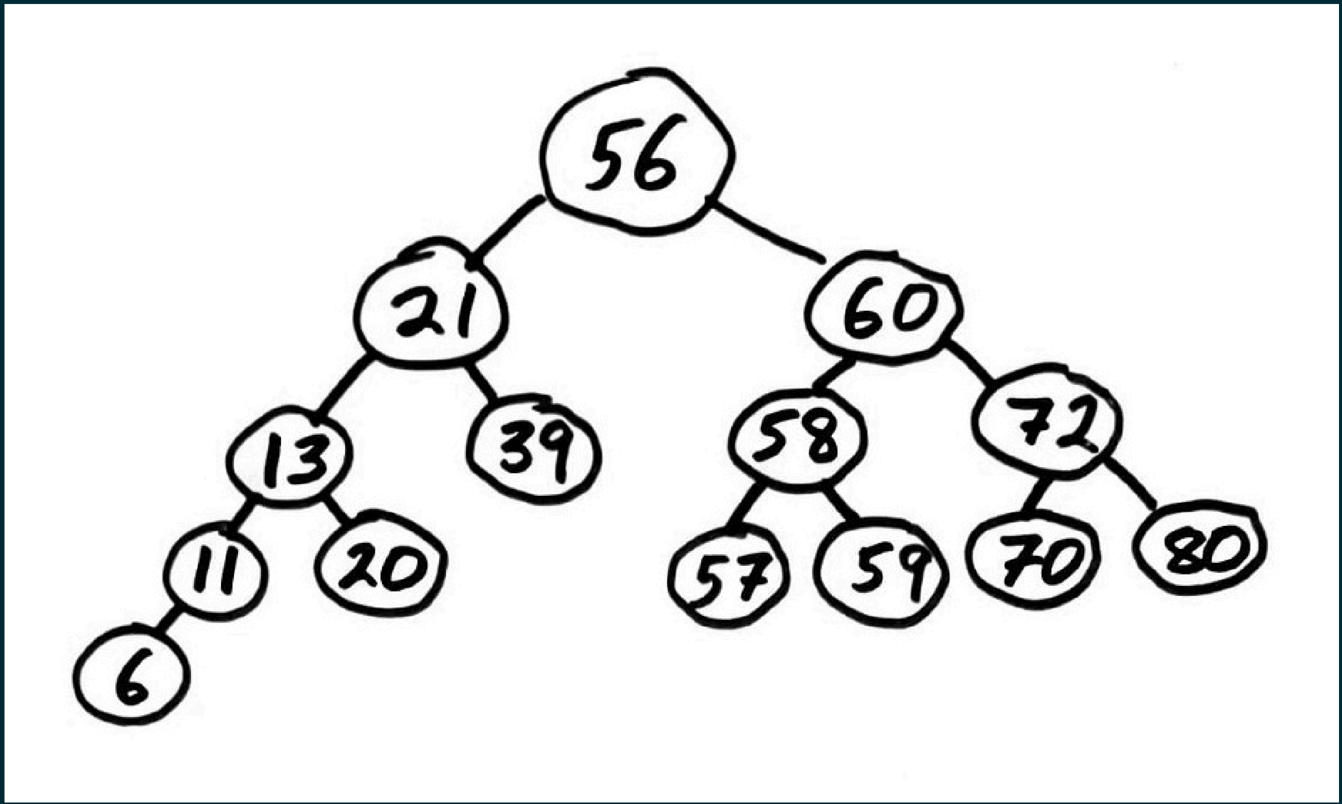
This turns out to be a very efficient data structure for many purposes. A lot of data can be accessed in a few steps from the root node.

BINARY SEARCH TREE

Stores numbers or other objects allowing comparison as node values. Enforce the rule: Each node's left child and its descendents are smaller (or equal) to the node. Each node's right child and its descendents are greater than the node.



This allows a natural way to check if a value is present with a game of "too high / too low".



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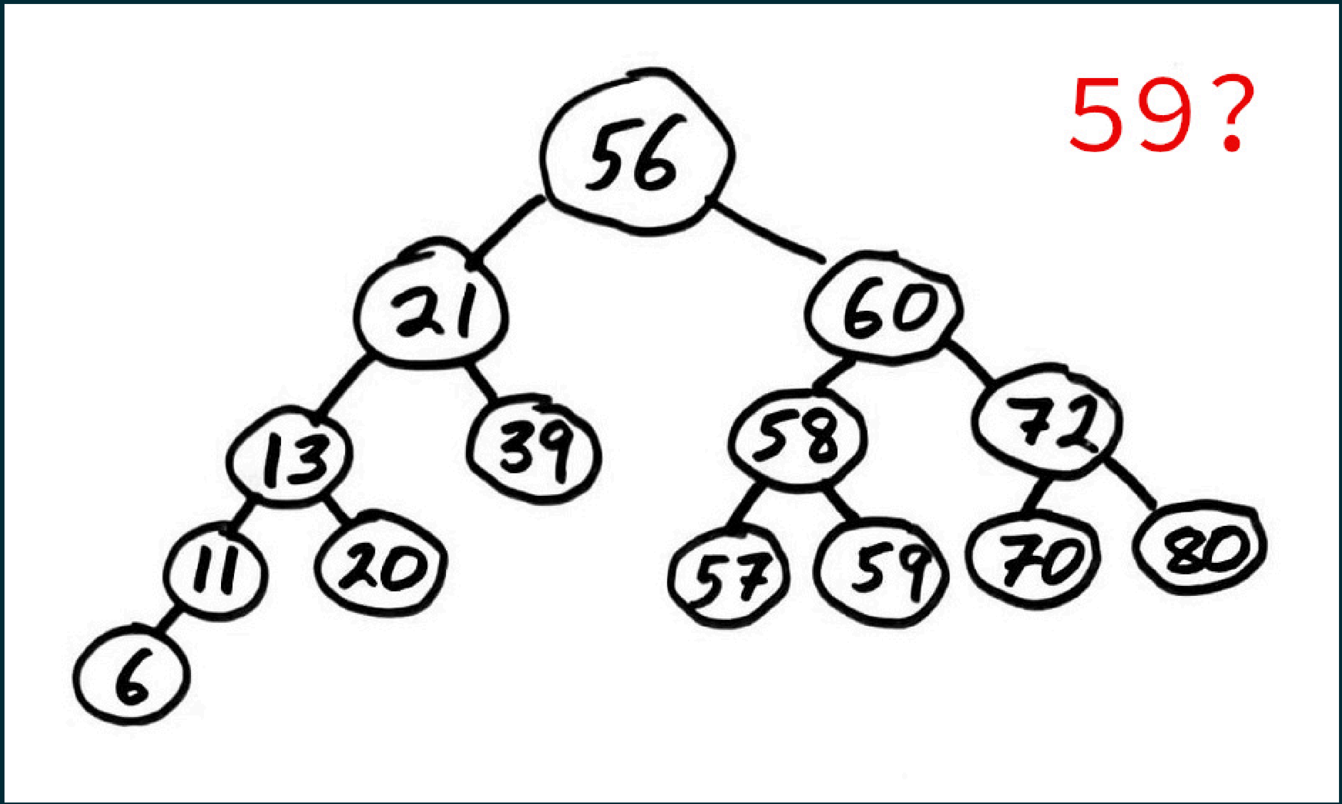

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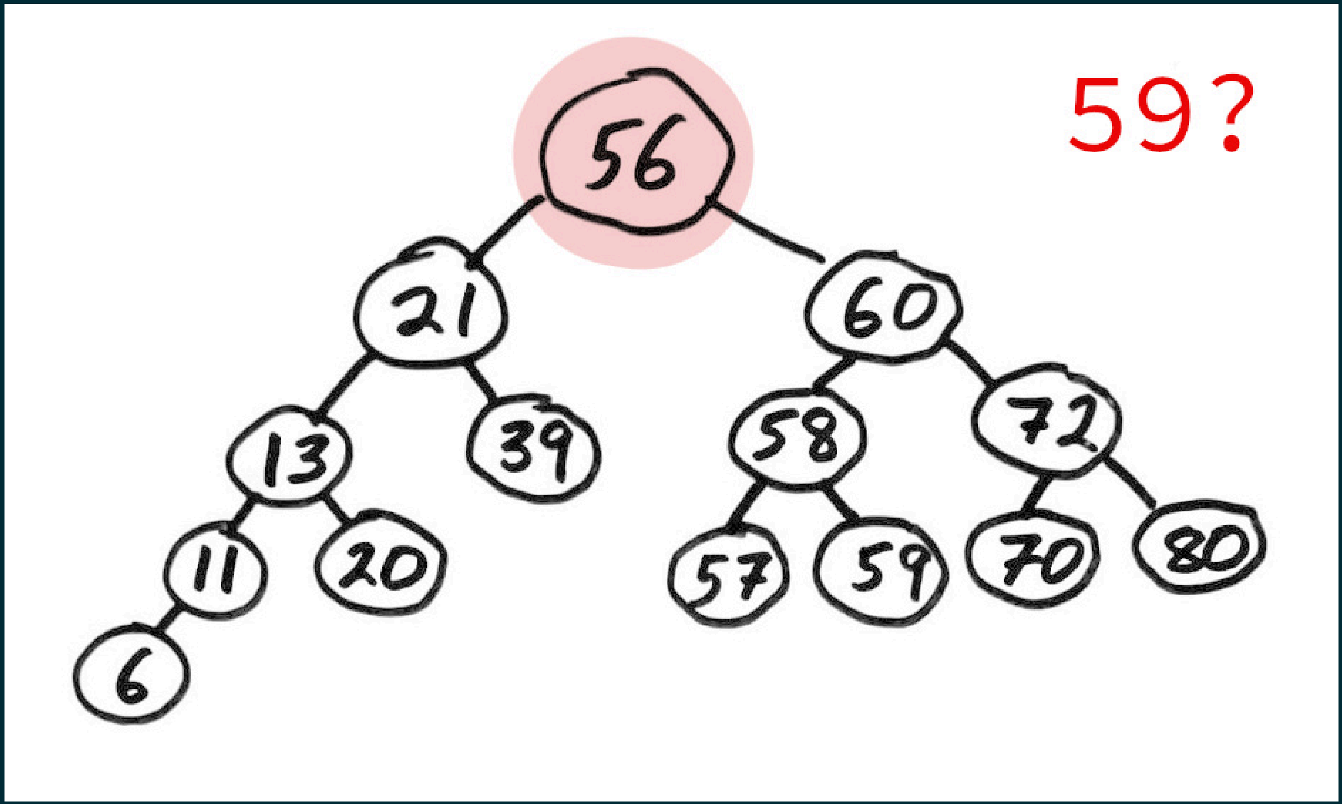

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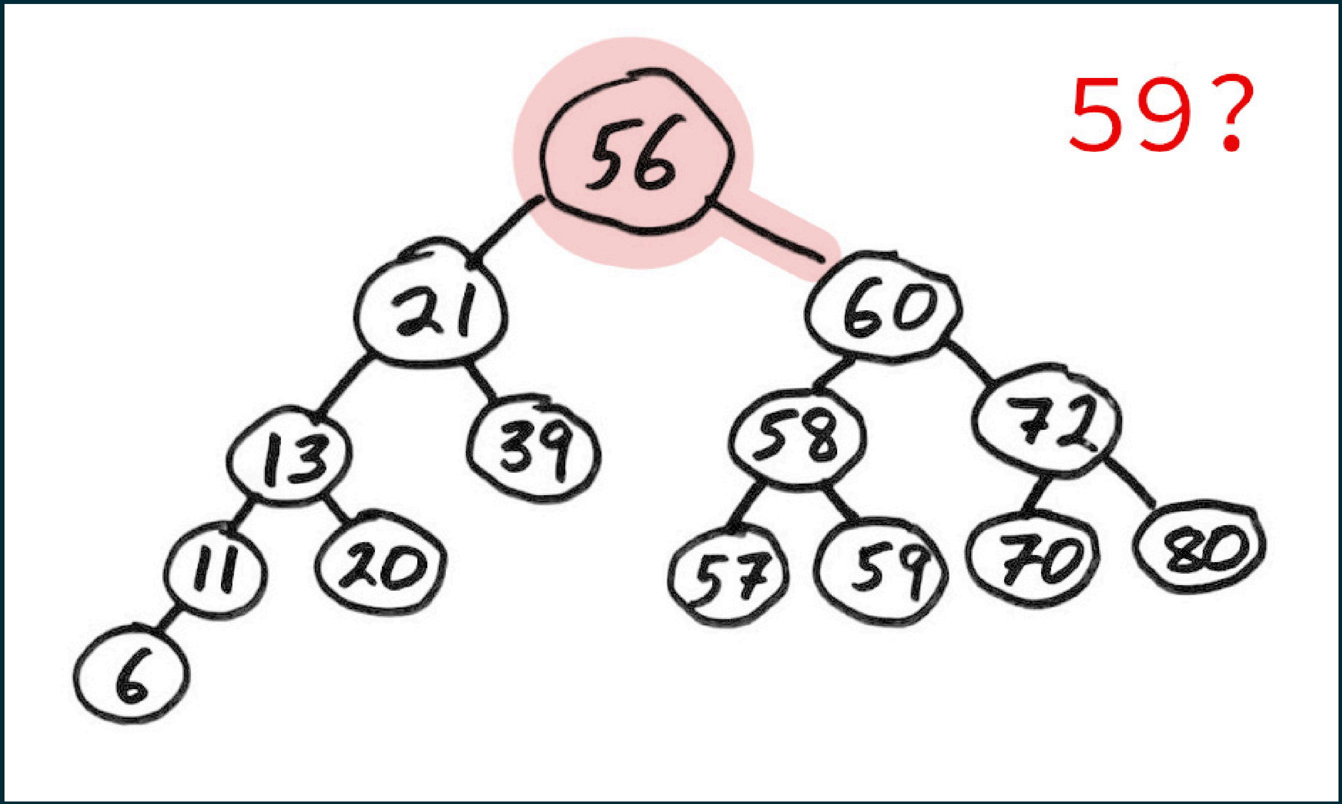

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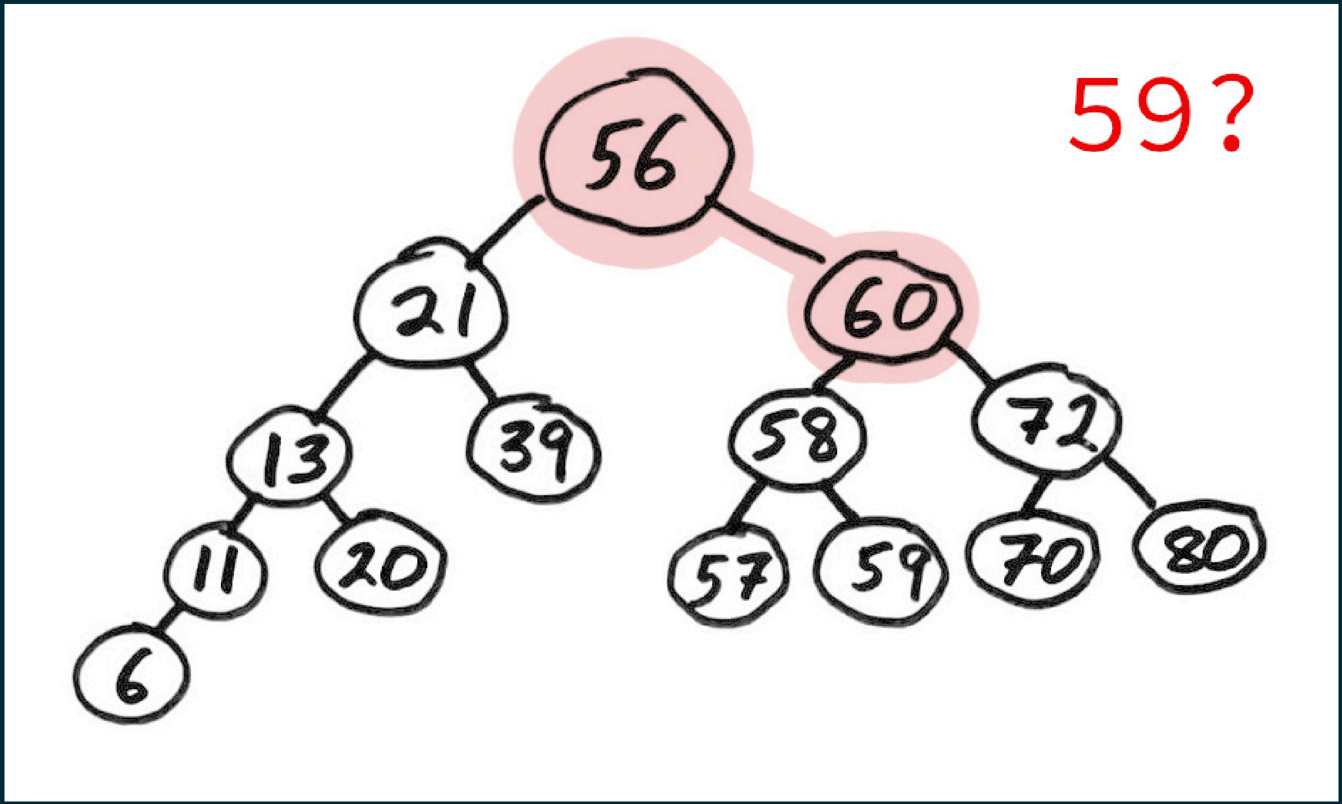

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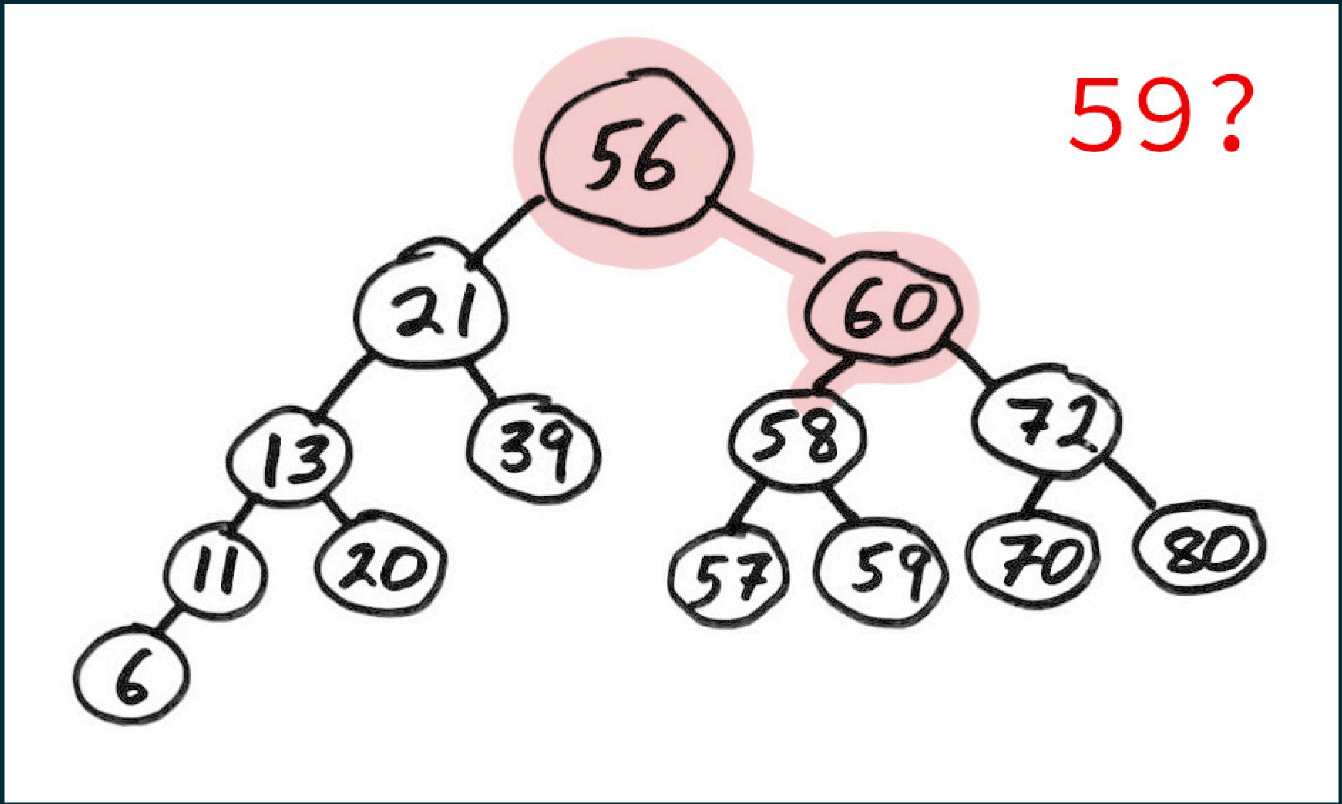

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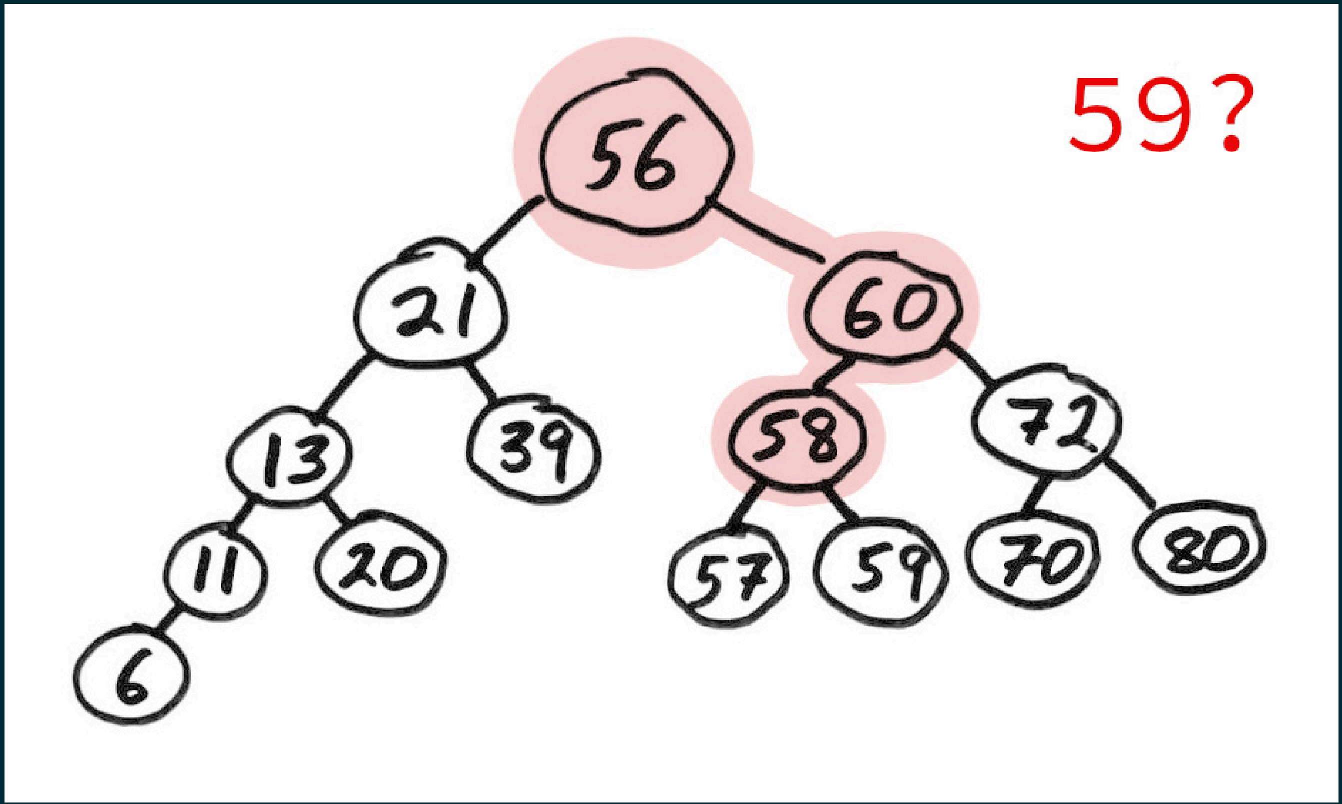

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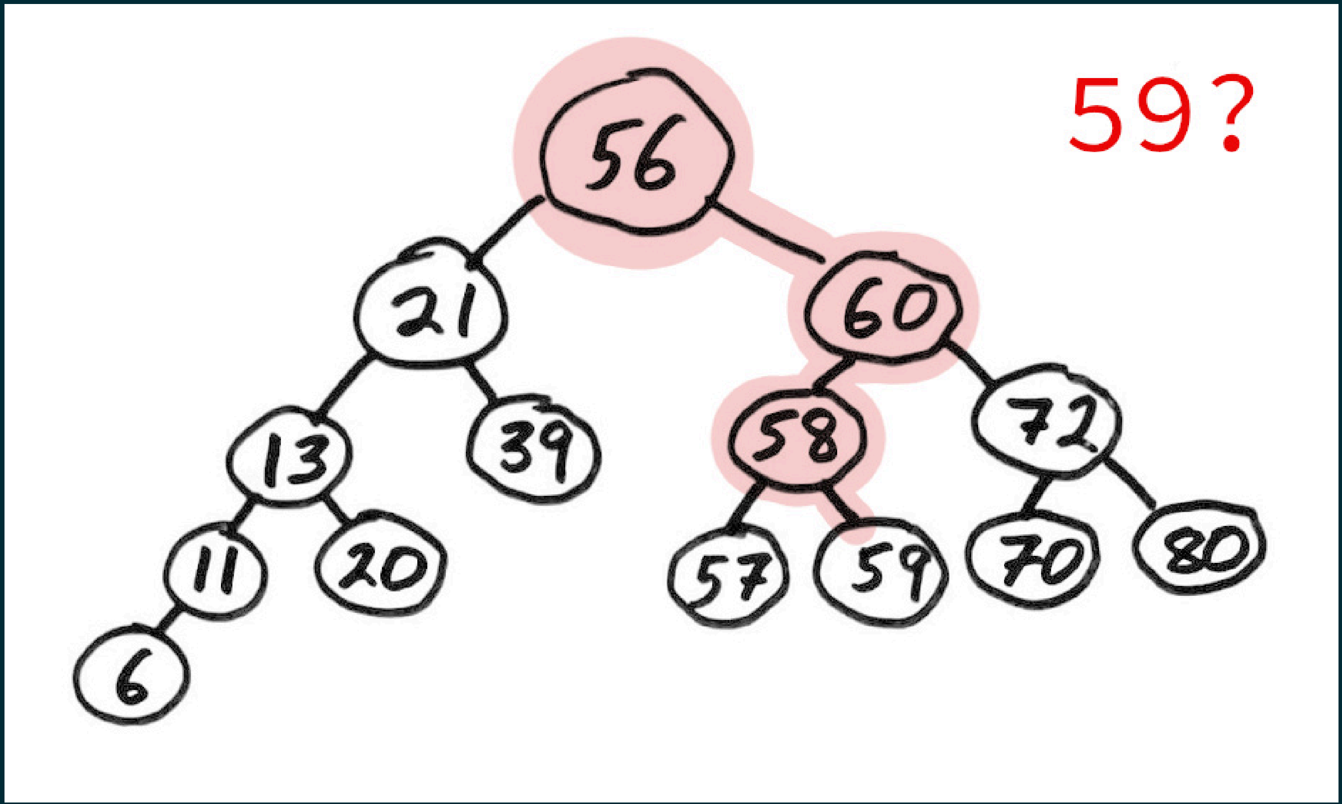

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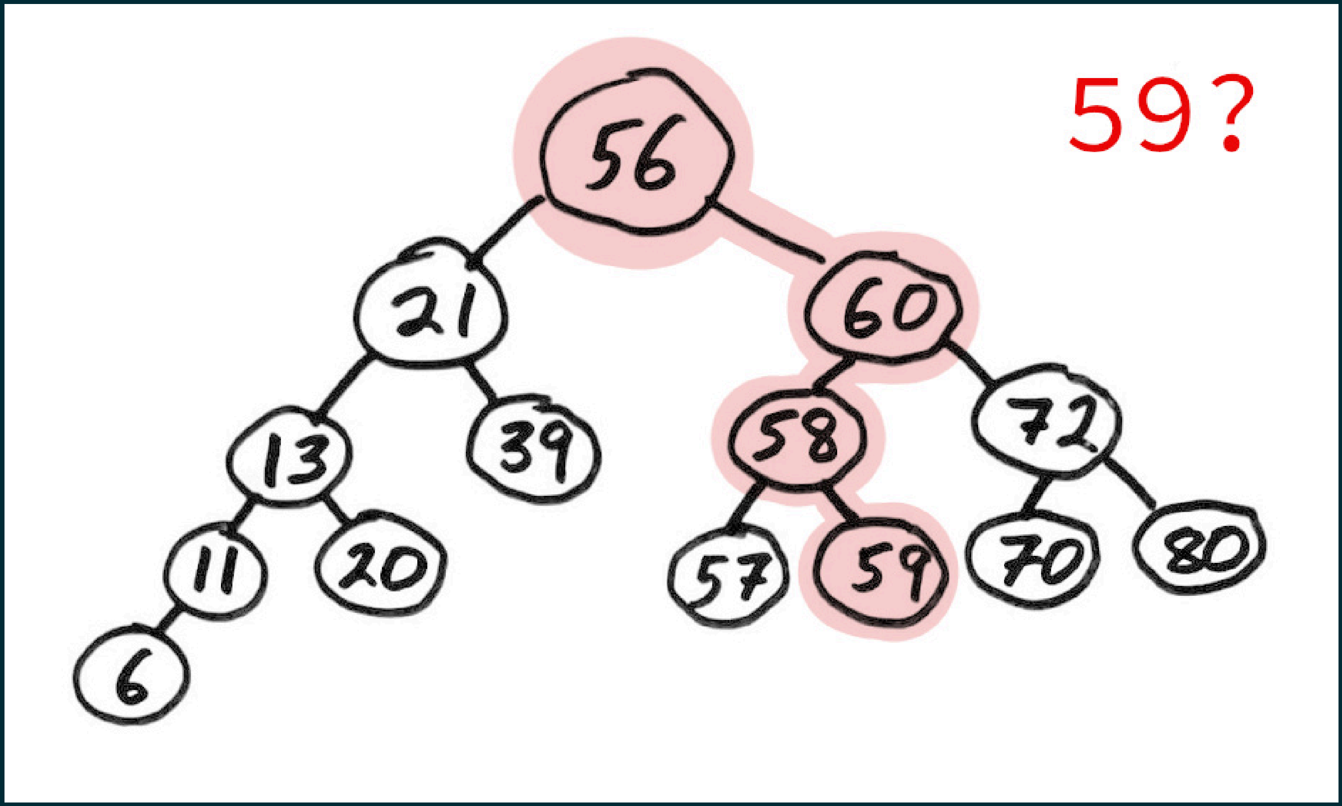

REFERENCES

- In optional course texts:
  - *Problem Solving with Algorithms and Data Structures using Python* by Miller and Ranum, discusses binary trees in Chapter 7.
- Elsewhere:
  - Cormen, Leiserson, Rivest, and Stein discusses graph theory and trees in Appendices B.4 and B.5, and binary search trees in Chapter 12.

REVISION HISTORY

- 2021-02-24 Correct publication date and fix BST definition slide
- 2021-02-24 Initial publication