LECTURE 16

MERGESORT

MCS 275 Spring 2021

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LECTURE 16: MERGESORT

Course bulletins:

- Starting with Quiz 6, you will have 48 hours for quizzes (Noon Sunday to Noon Tuesday).
- Project 2 description updated with sample data and modules policy.
- Project 2 due 6pm CST Friday, February 26.
- Check out the recursion sample code.
- Worksheet 7 will explore recursive maze solver / generator in more depth.

PLAN

- Discuss the theory of
  - Divide and conquer
  - Sorting
  - Mergesort
- Implement mergesort

DIVIDE AND CONQUER

A strategy that often involves recursion.

- **Split** a problem into parts.
- **Solve** for each part.
- **Merge** the partial solutions into a solution of the original problem.

Not always possible or a good idea. It only works if merging partial solutions is easier than solving the entire problem.

COMPARISON SORT

Suppose you have a list of objects that can be compared with $==$, $>$, $<$.

You'd like to reorder them in increasing order.

This problem is called **comparison sort**. There are many solutions.

MERGESORT

A divide-and-conquer solution to comparison sort.

It is a fast solution, often used in practice.

Key: It is pretty easy to take two sorted lists and merge them into a single sorted list.

So, let's divide our list into halves, sort each one (recursively), then merge them.

Now we'll formalize this.

Algorithm `mergesort`:

Input: list `L` whose elements support comparison.

Goal: reorder the elements of `L` in place to achieve sorted order.

1. If `L` has 0 or 1 elements, it is already sorted. Do nothing.
2. Otherwise, copy the first half of `L` into a new list `L1`, and the rest into `L2`.
3. Use recursive calls to sort `L1` and `L2` (in place).
4. Use `merge_sorted_lists` to merge `L1` and `L2` into `L`.

MERGESORT EXAMPLE

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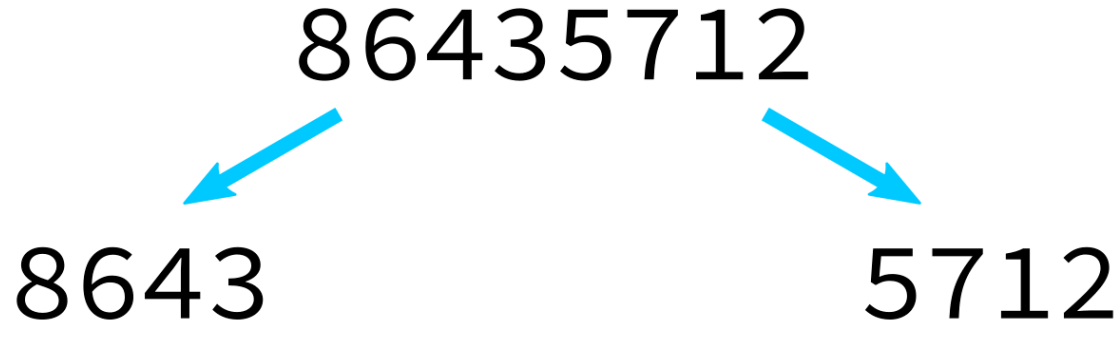
MERGESORT EXAMPLE



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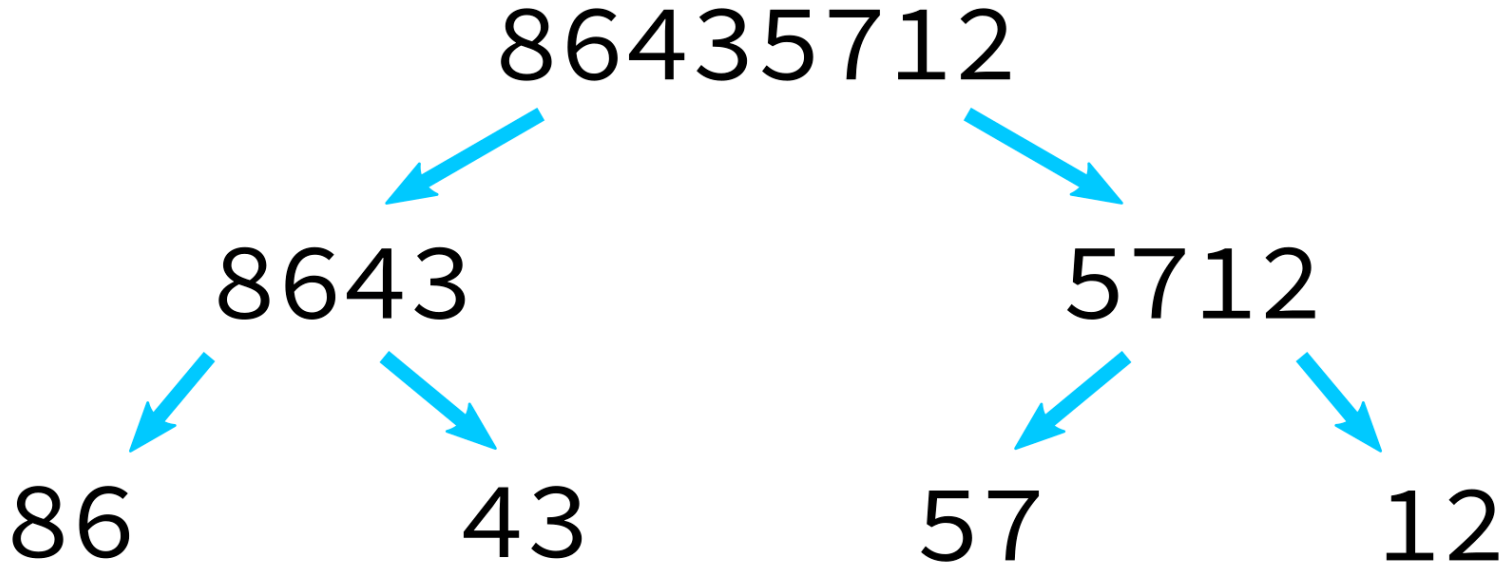

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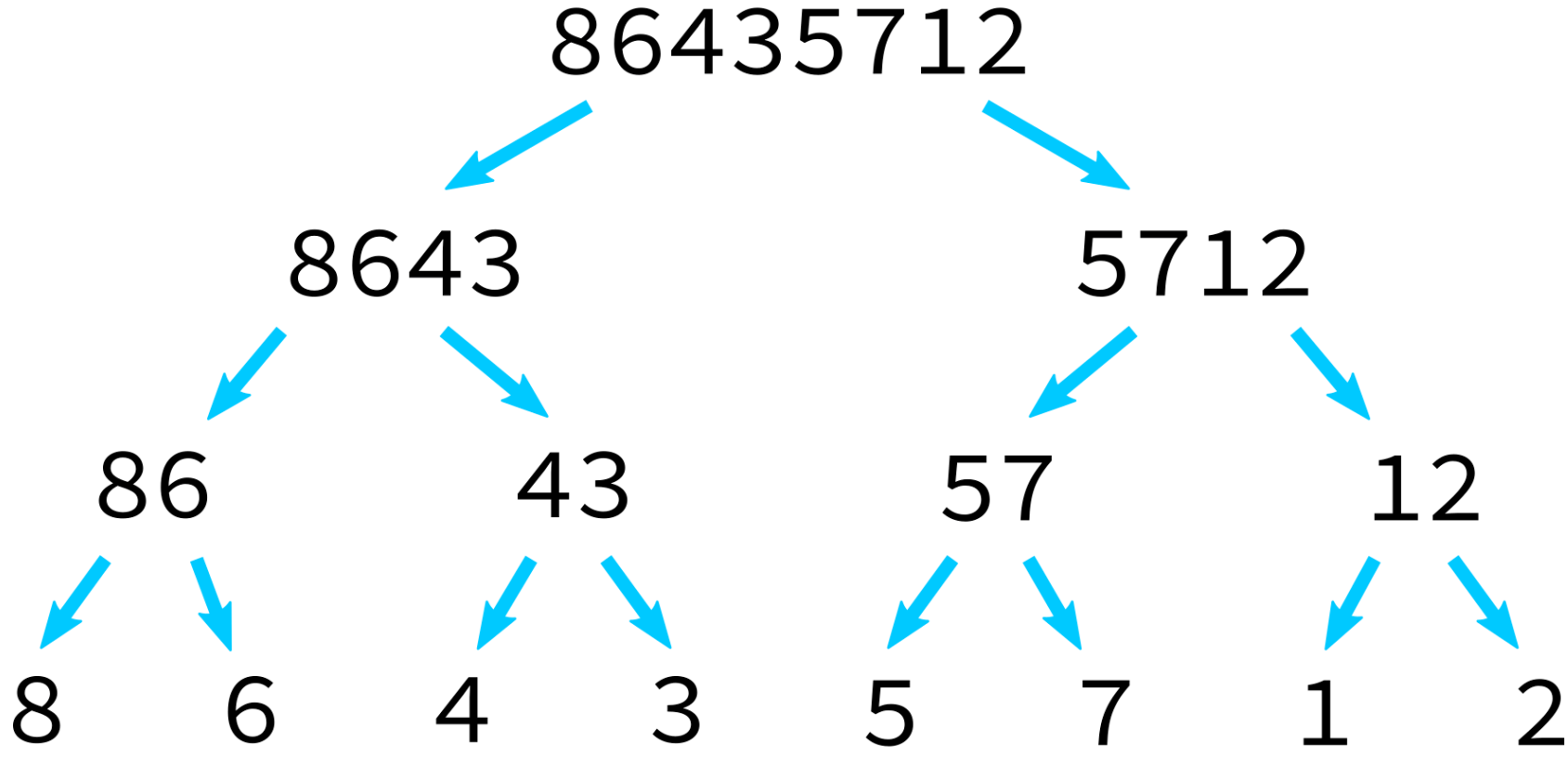

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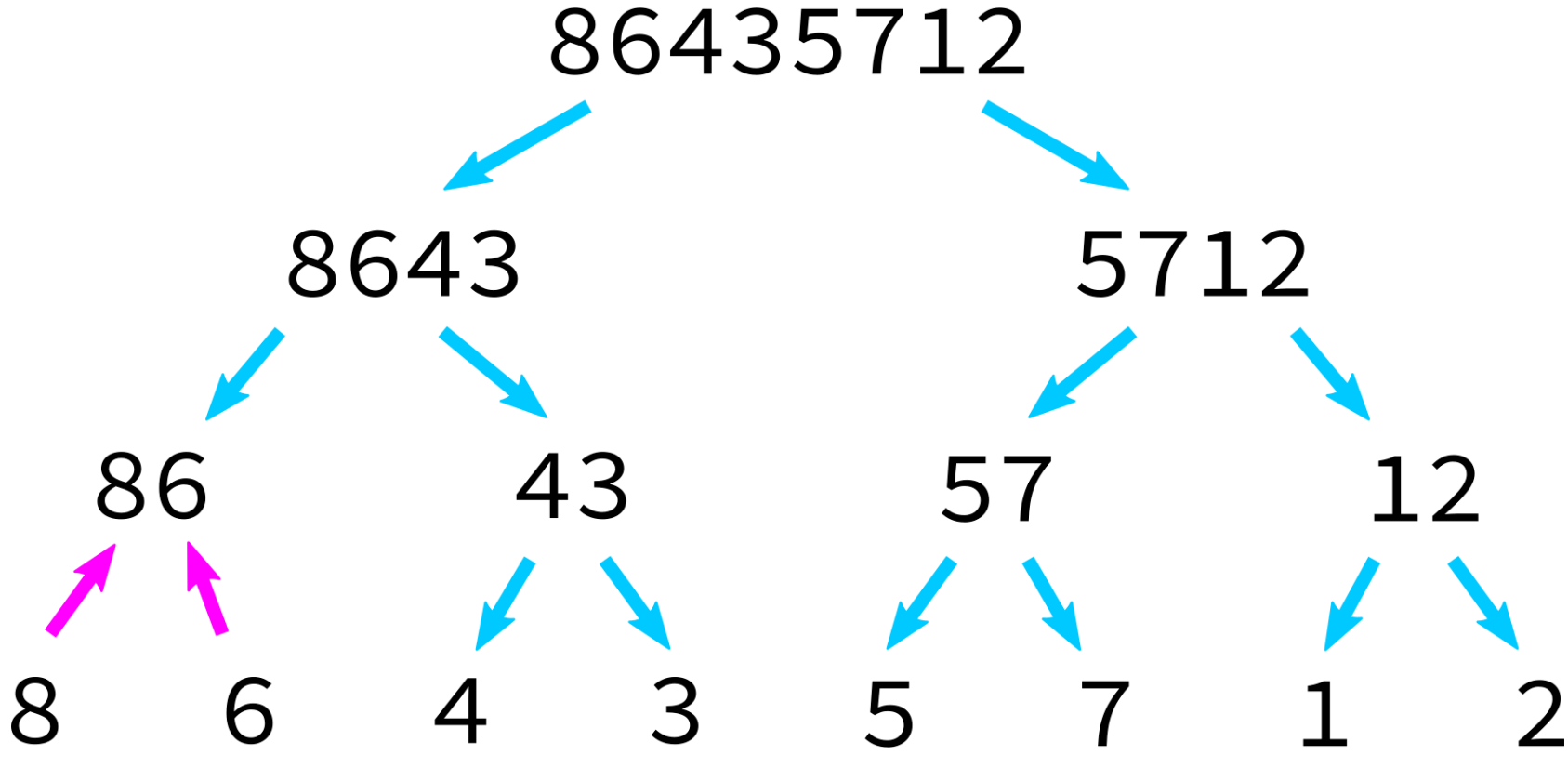

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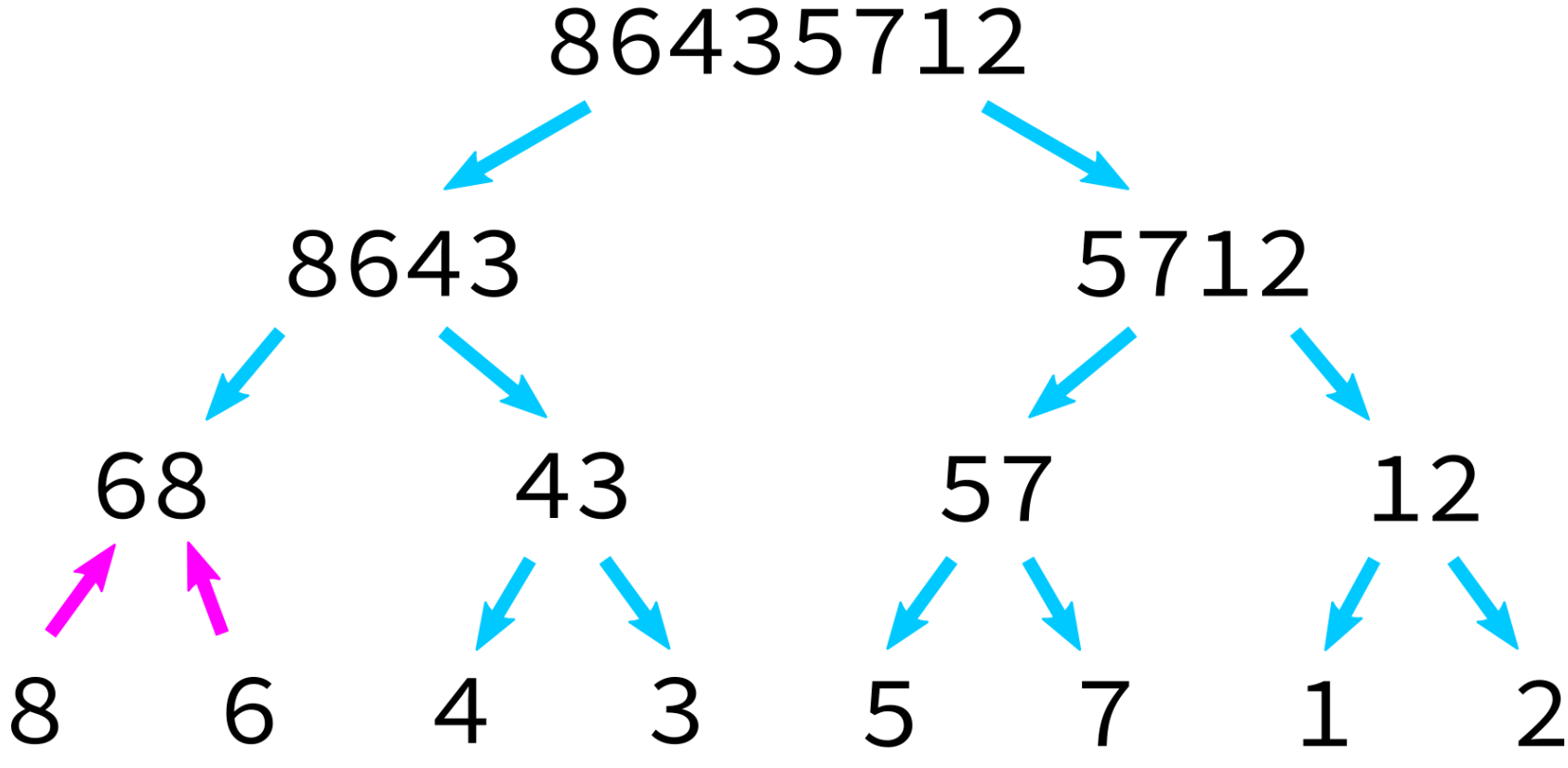

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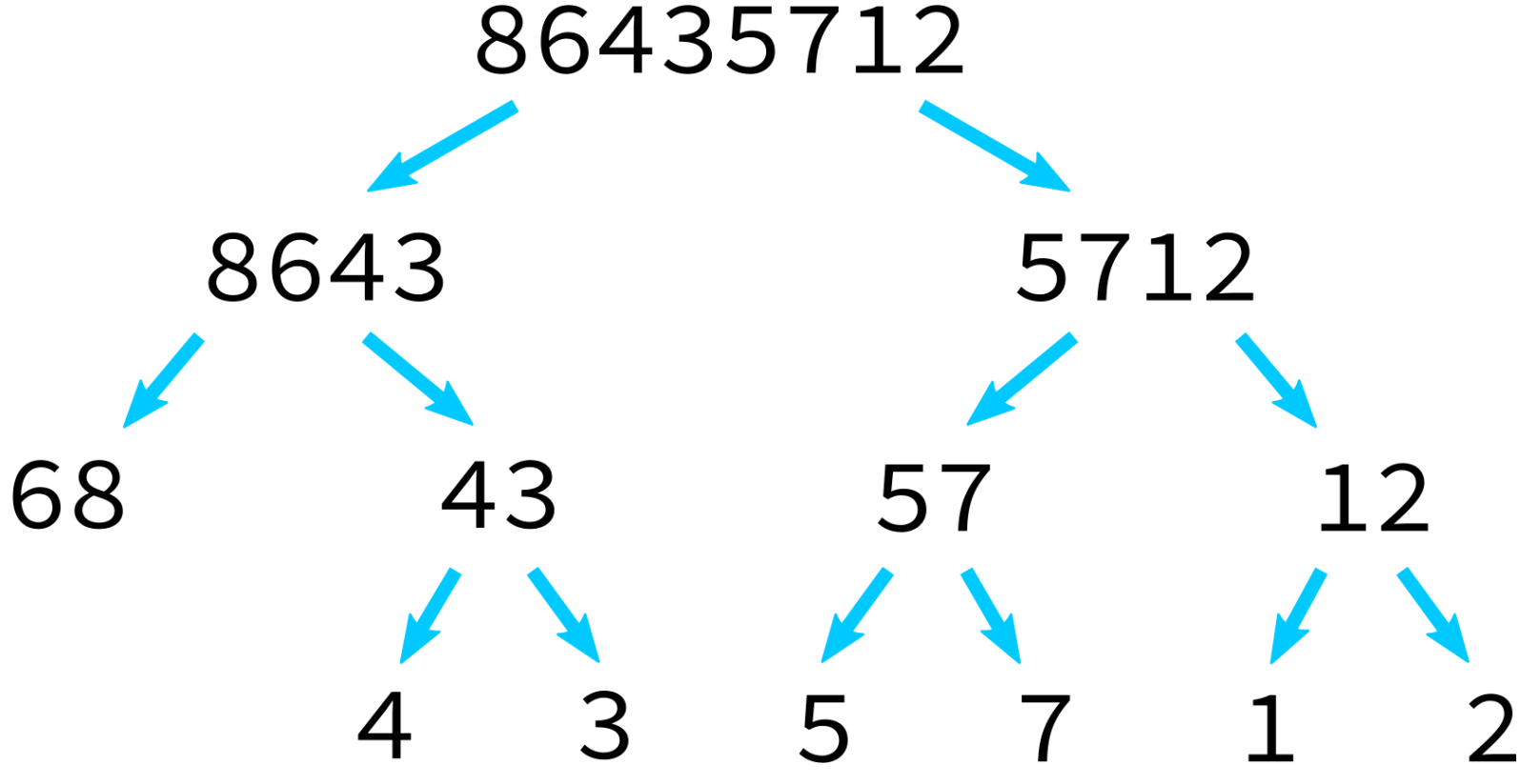

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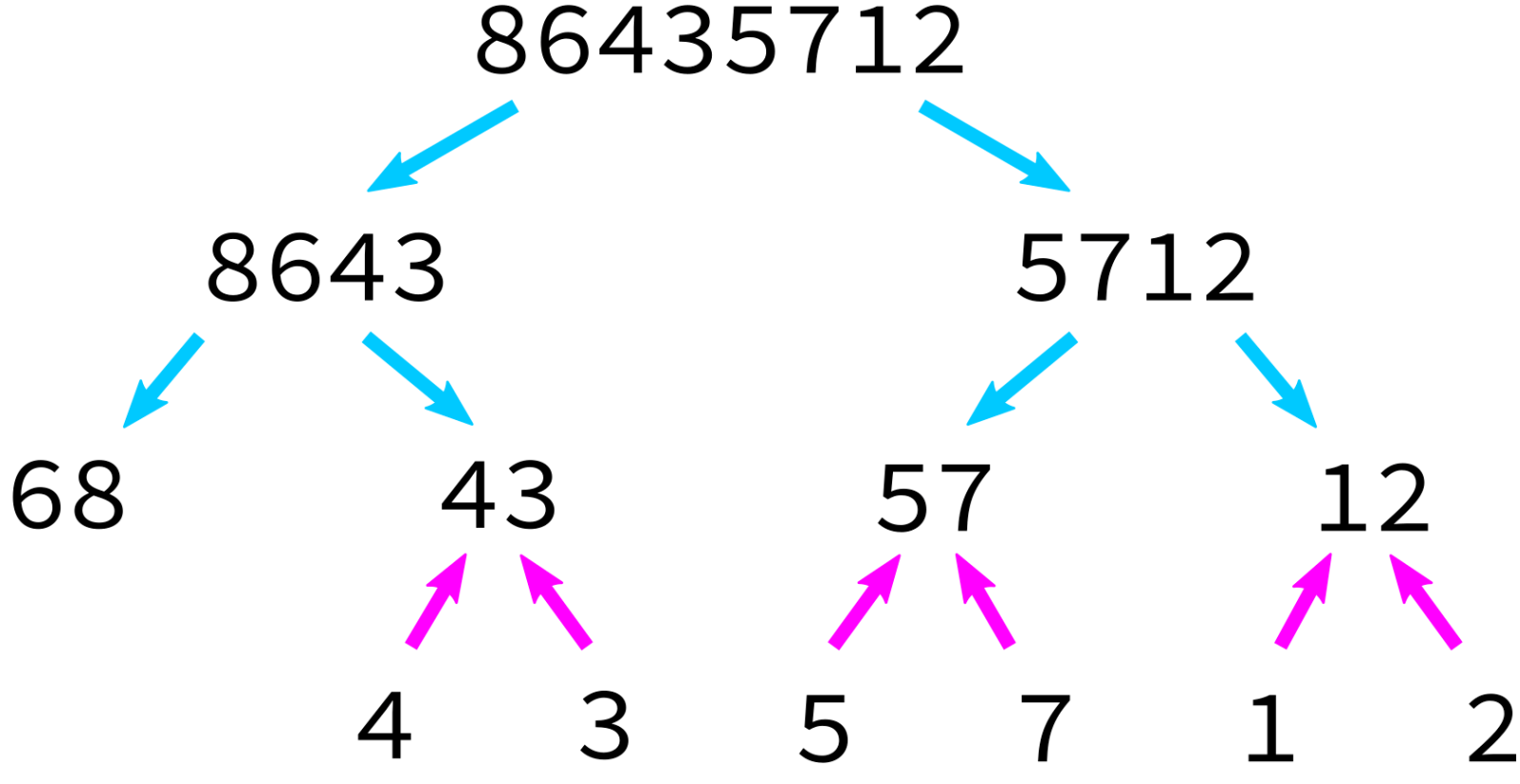

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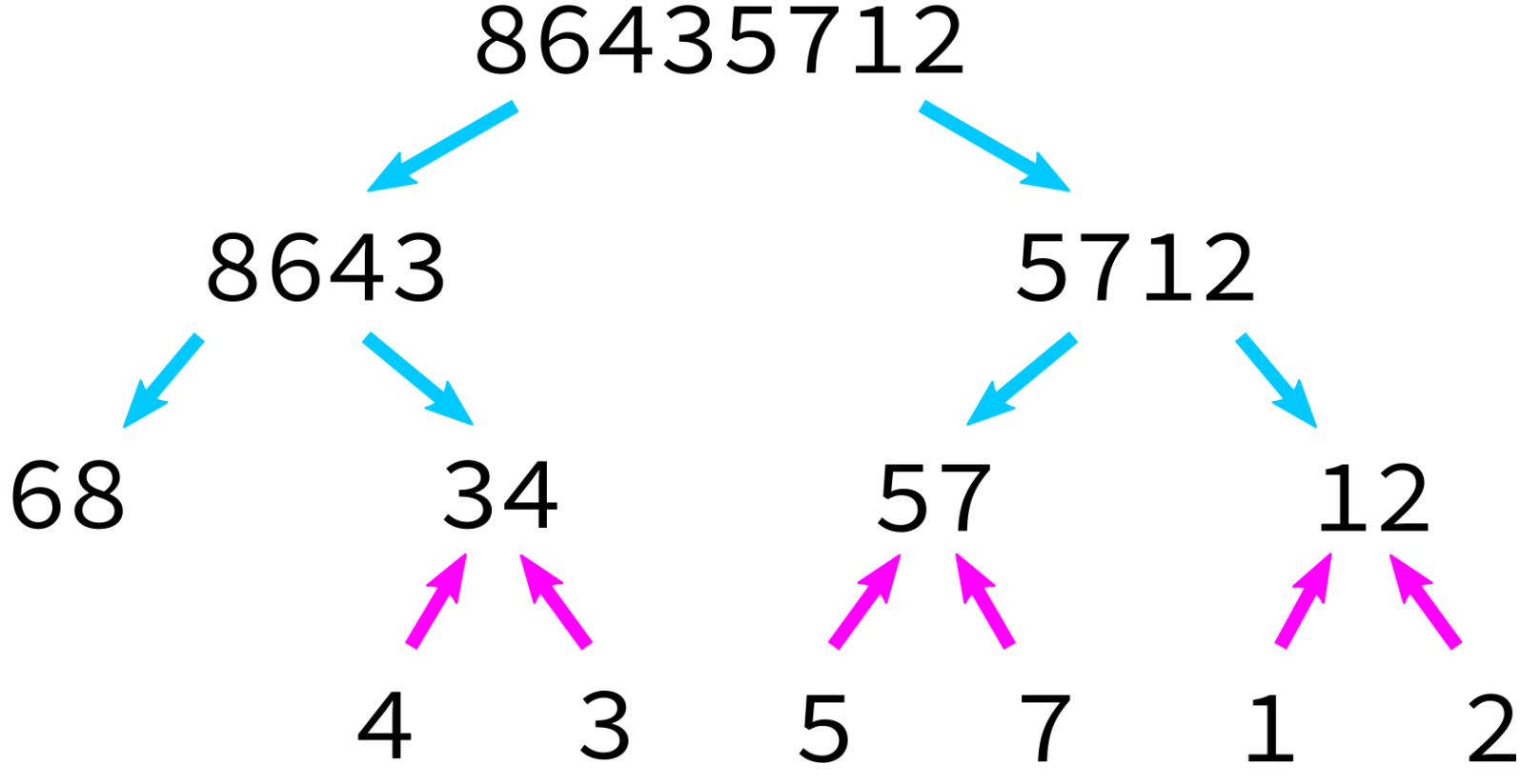

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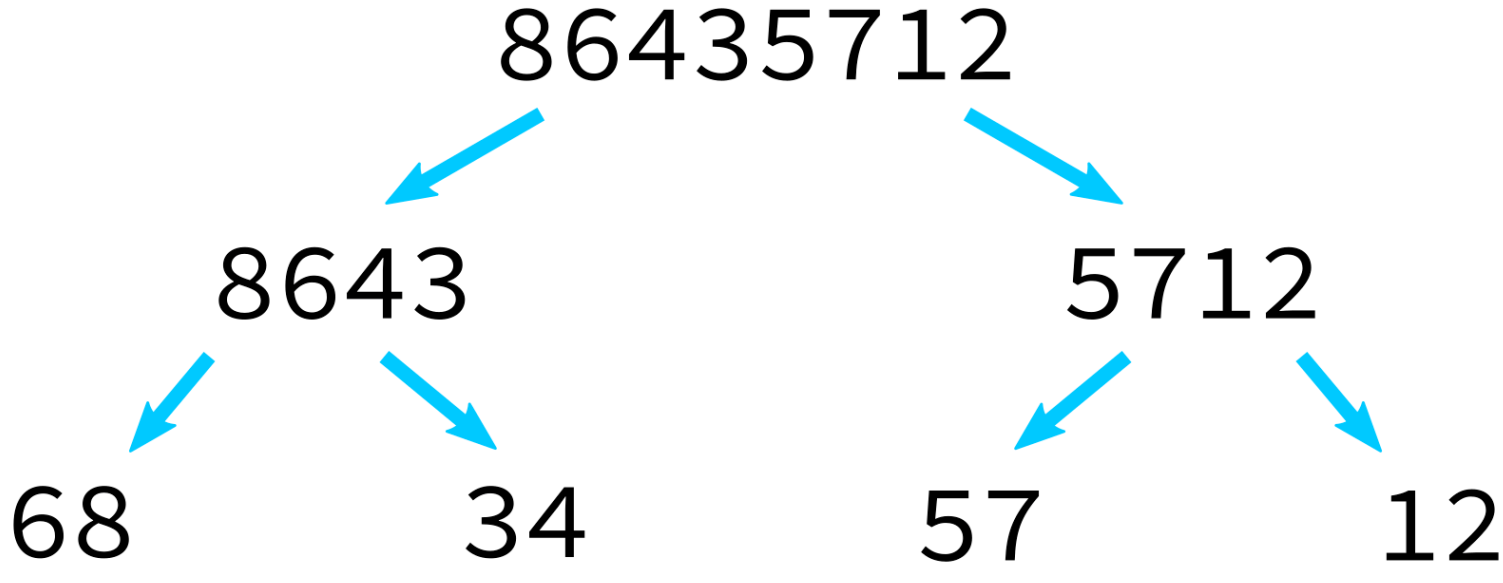

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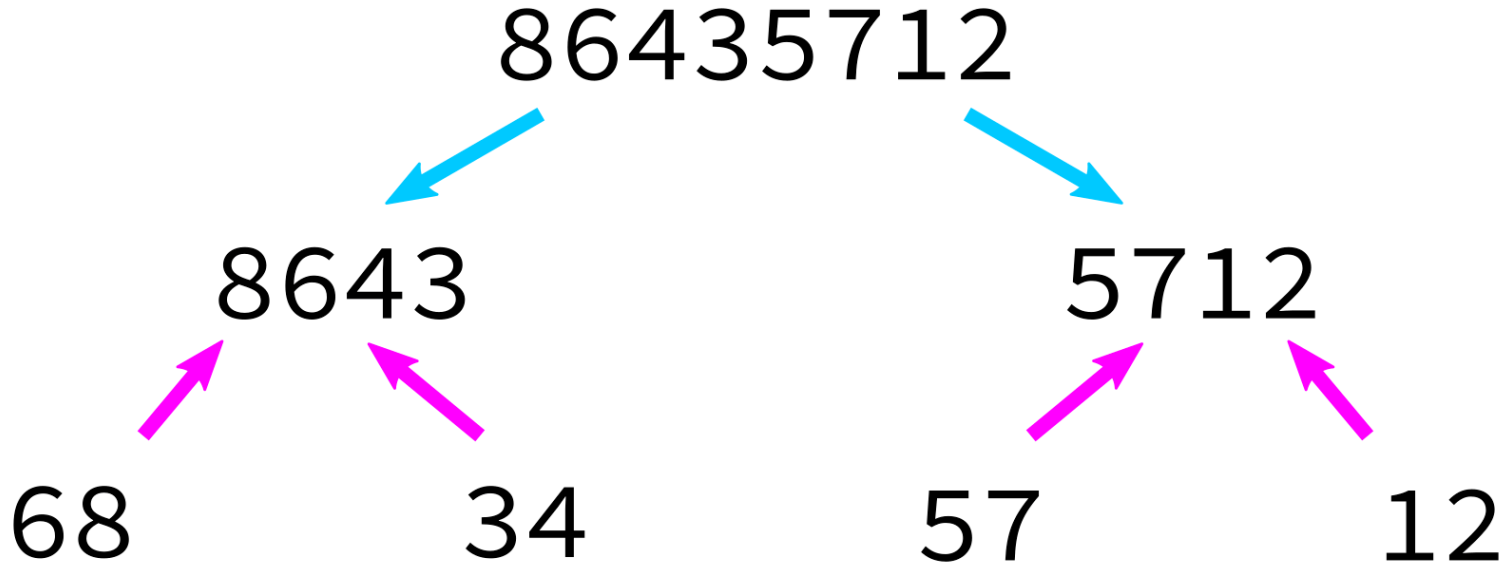

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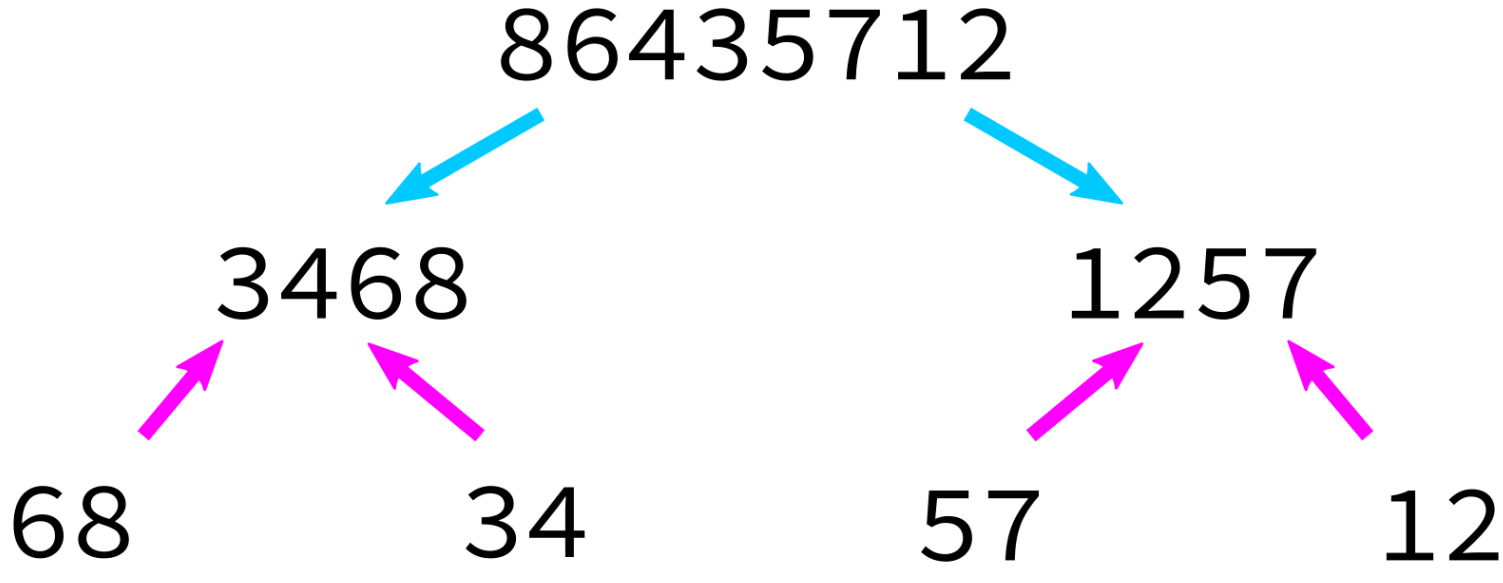

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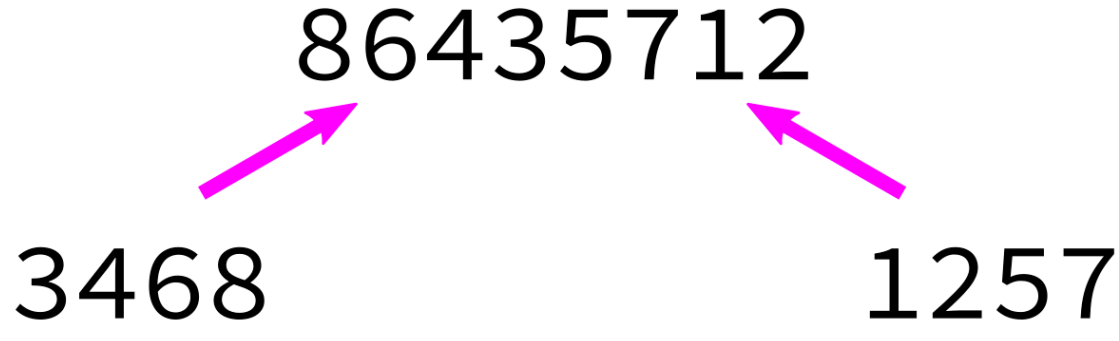

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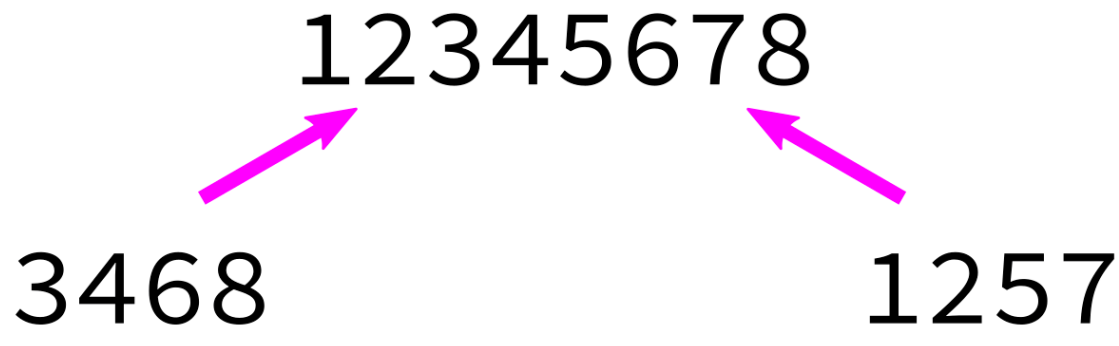

MERGESORT EXAMPLE

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BUT HOW TO MERGE?

This algorithm depends on having a function `merge_sorted_lists` that can merge to sorted lists into a single sorted list.

Algorithm `merge_sorted_lists`:

Input: sorted lists L1 and L2, and a list `L` of the proper length in which to store the results.

Goal: copy all elements of `L1` and `L2` into `L` in increasing order.

1. Make three integer variables `i1`, `i2`, `i` to keep track of current position in `L1`, `L2`, `L` respectively. Set all to zero.
2. While `i1 < len(L1)` and `i2 < len(L2)`, do the following:
   - Check which of `L1[i1]` and `L2[i2]` is smaller.
   - Store the smaller one in `L[i]`.
   - Increment whichever one of `i1`, `i2` was used.
   - Increment `i`
3. Copy any remaining portion of `L1` into `L`.
4. Copy any remaining portion of `L2` into `L`.

MERGING SORTED LISTS

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MERGING SORTED LISTS

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MERGING SORTED LISTS

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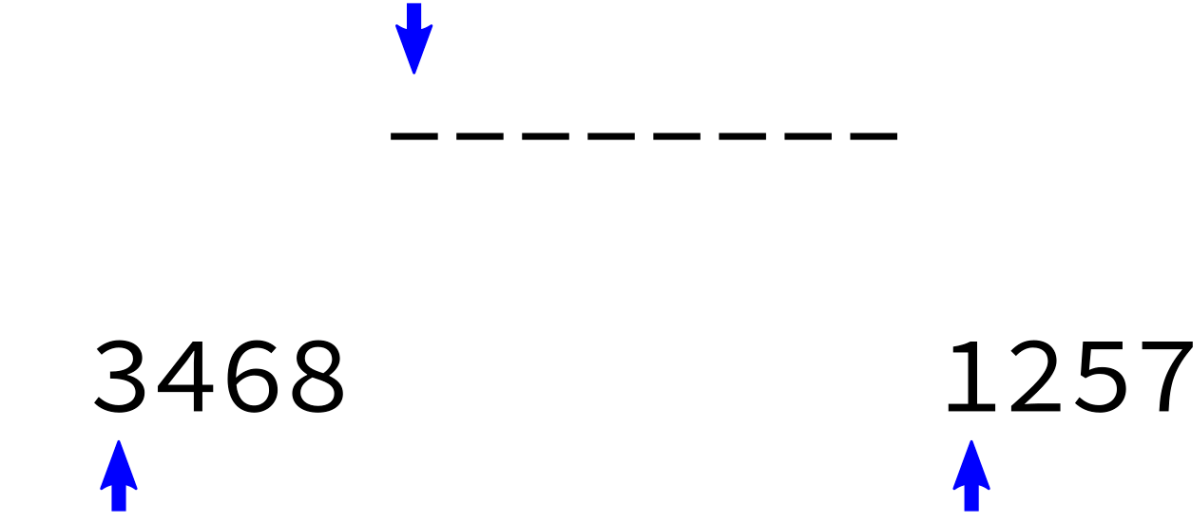

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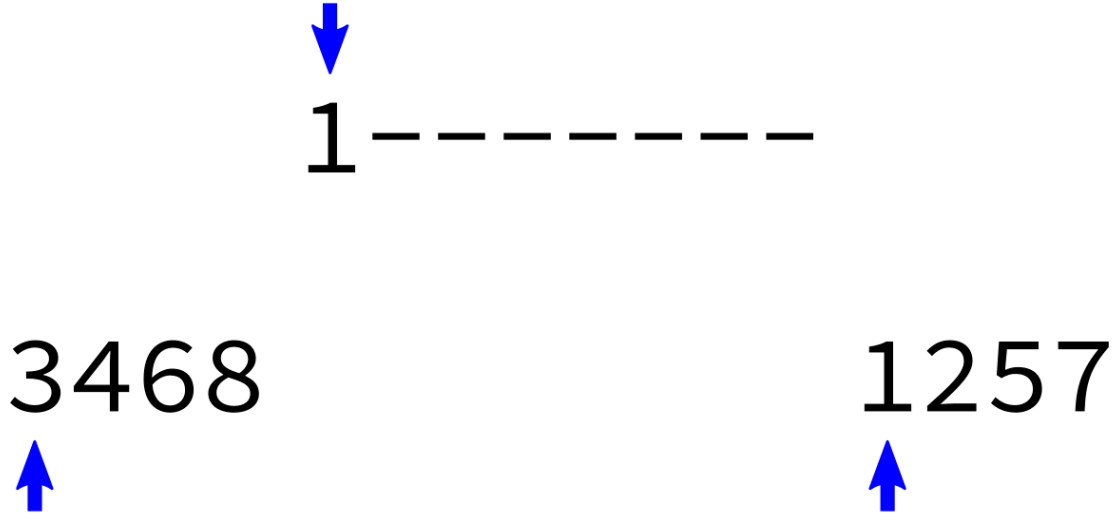

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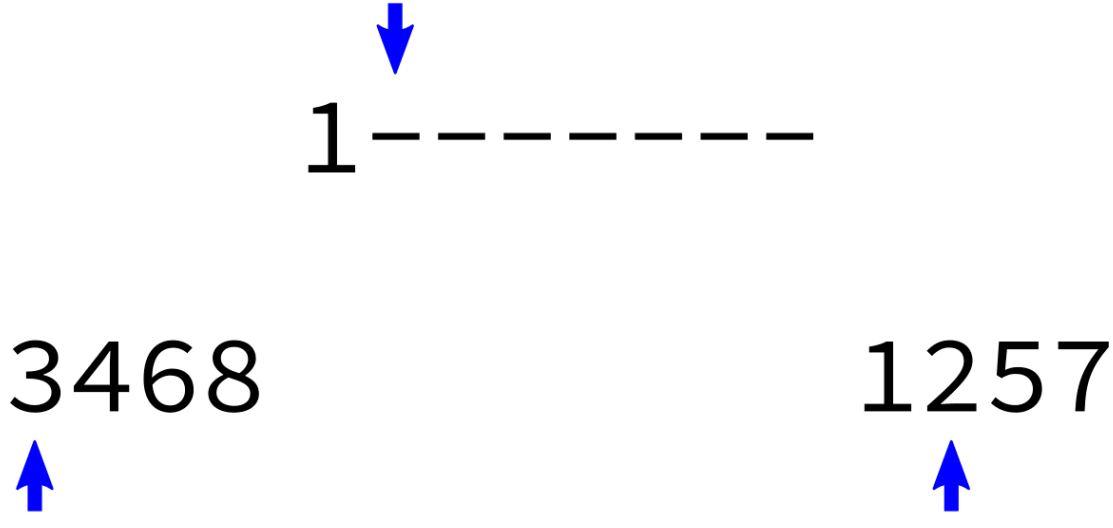

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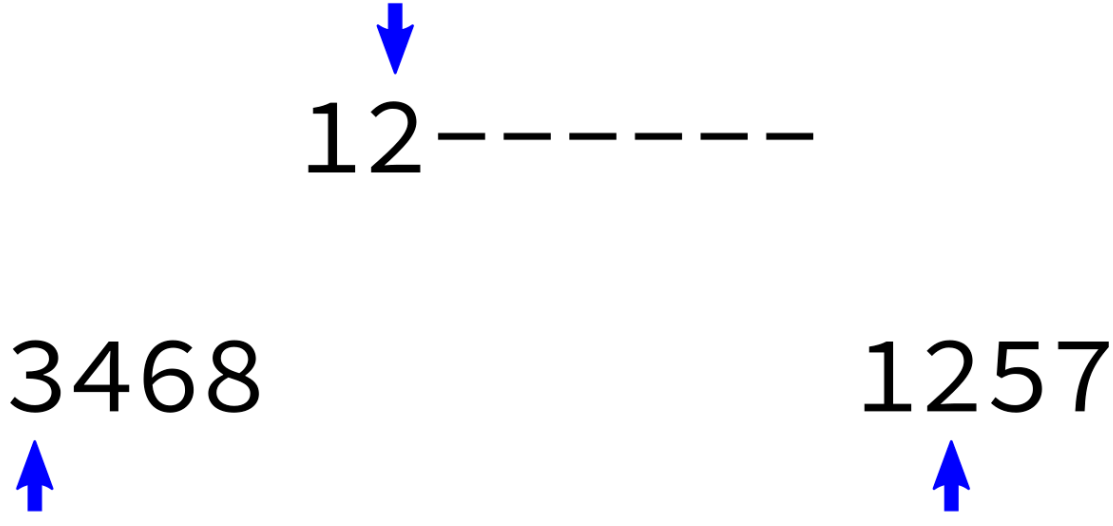

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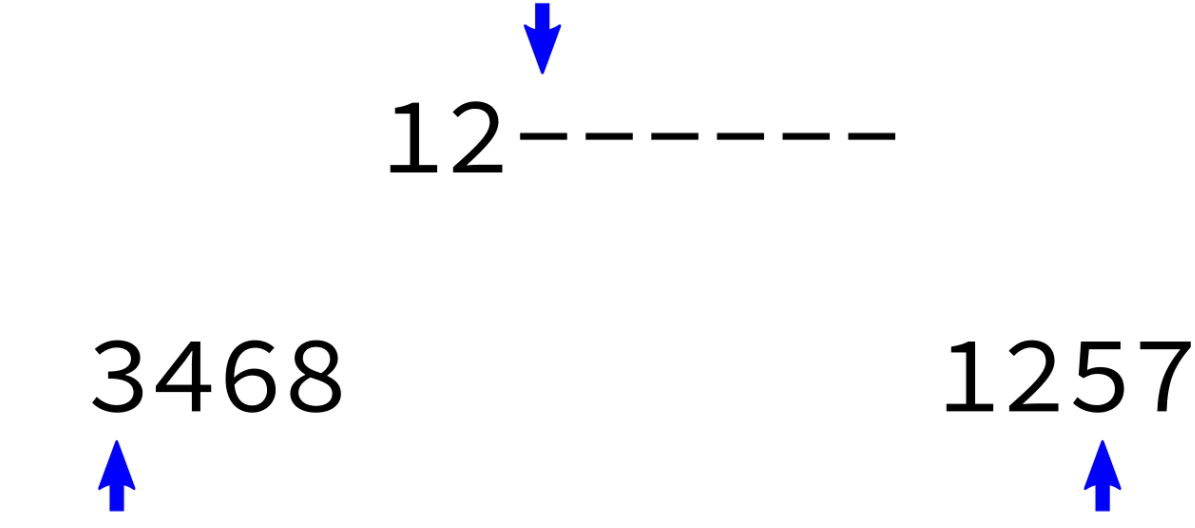

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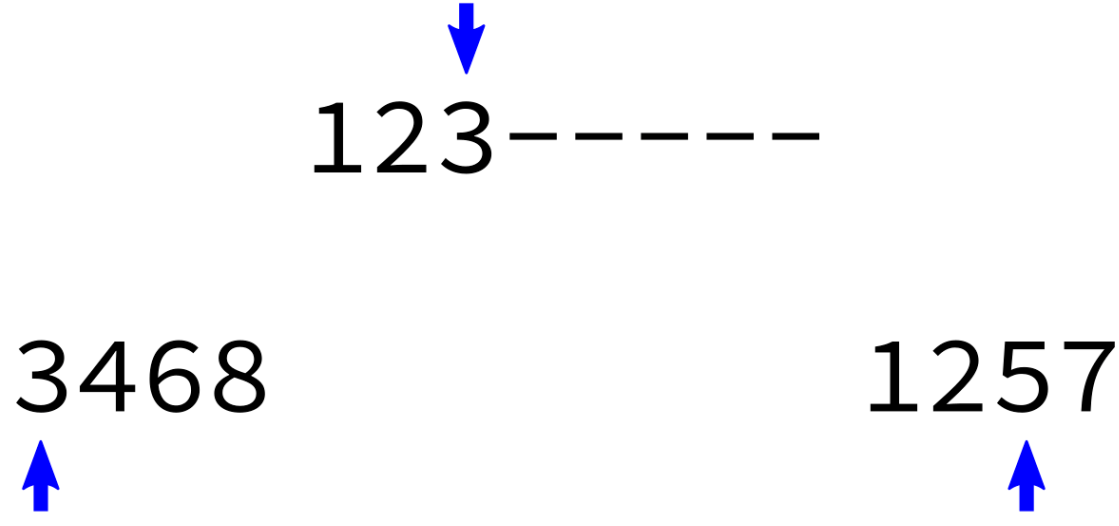

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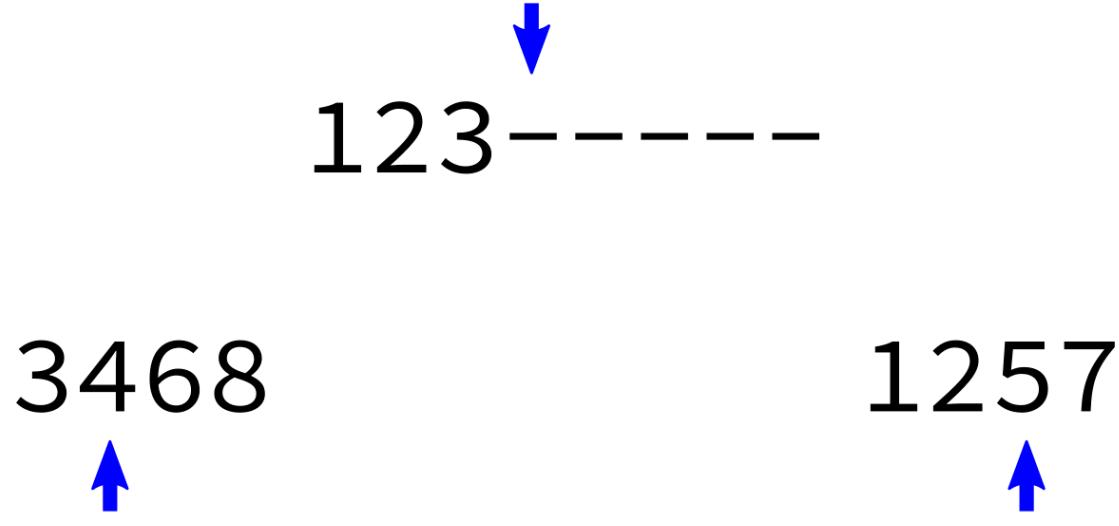

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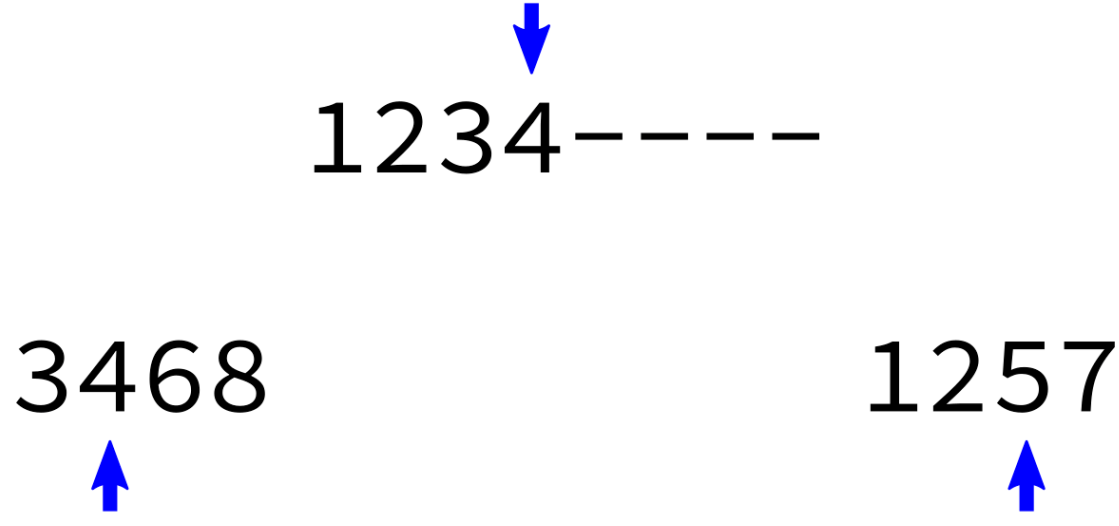

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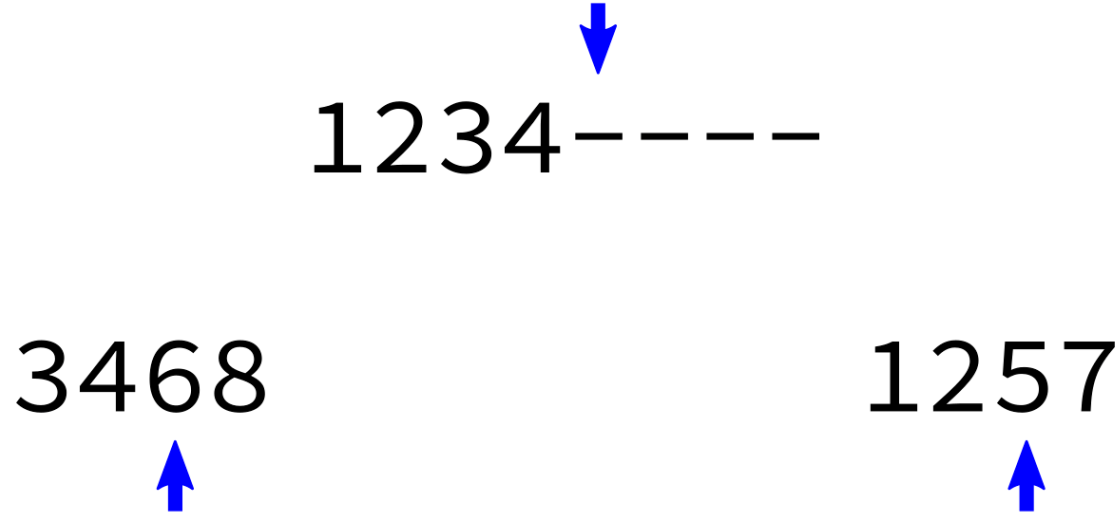

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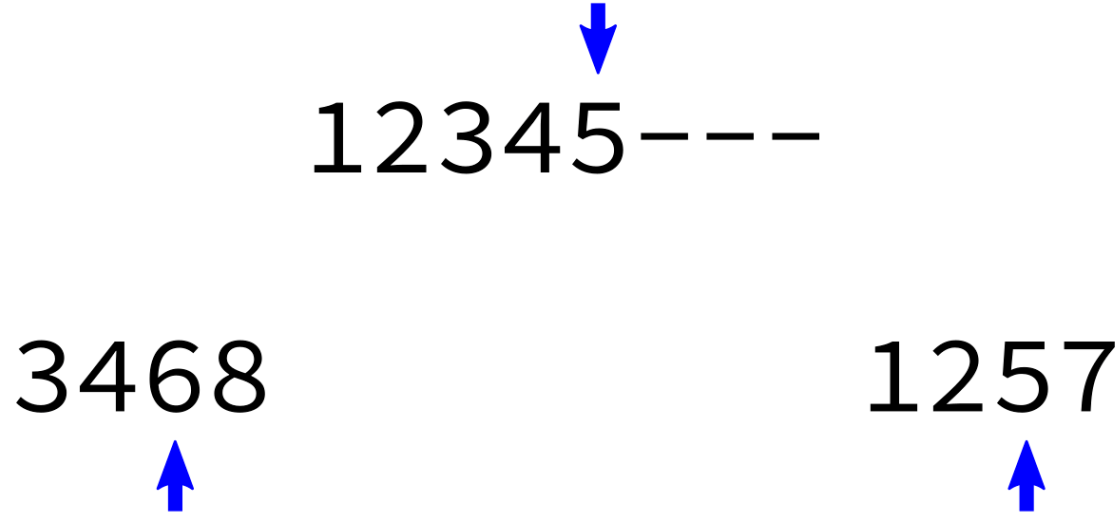

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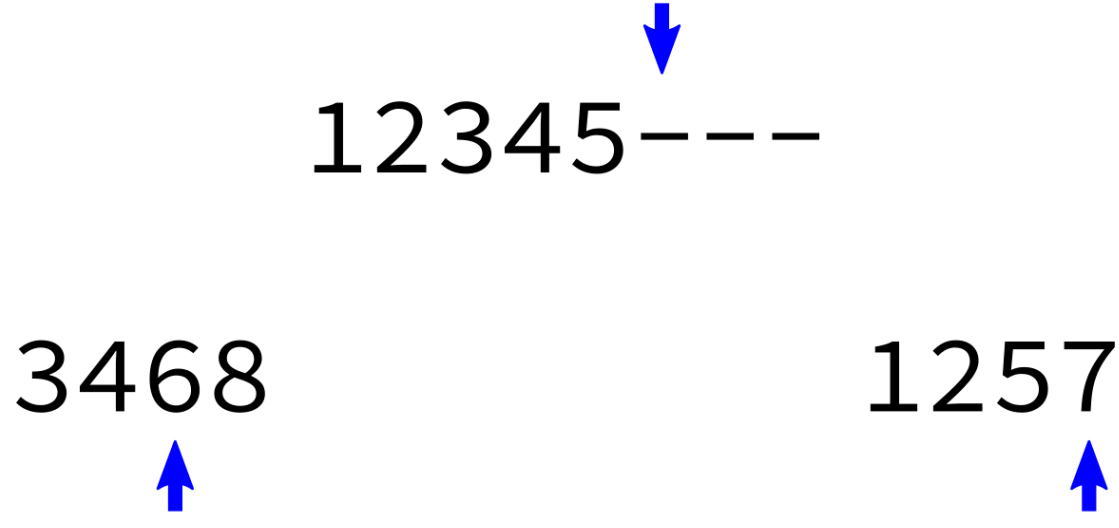

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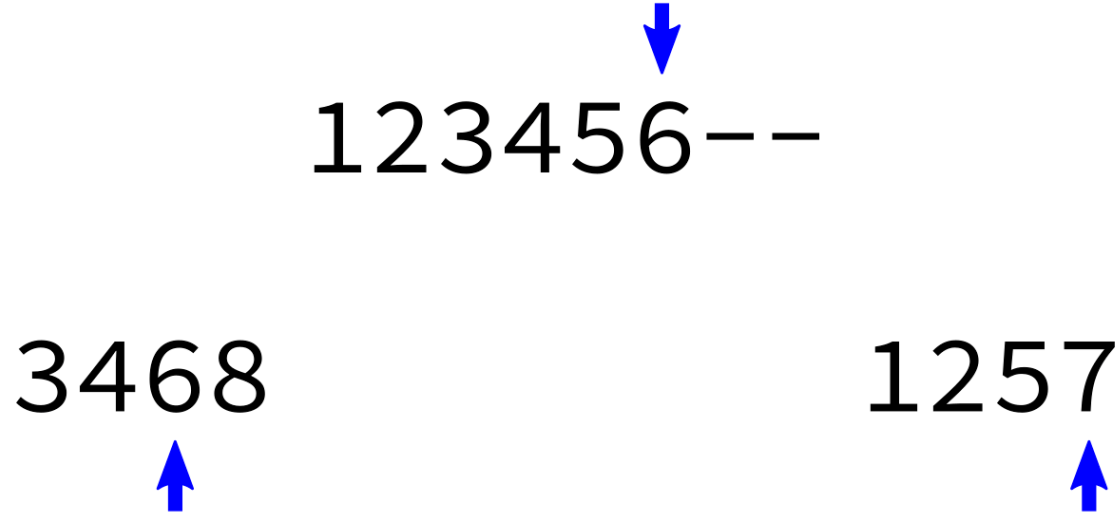

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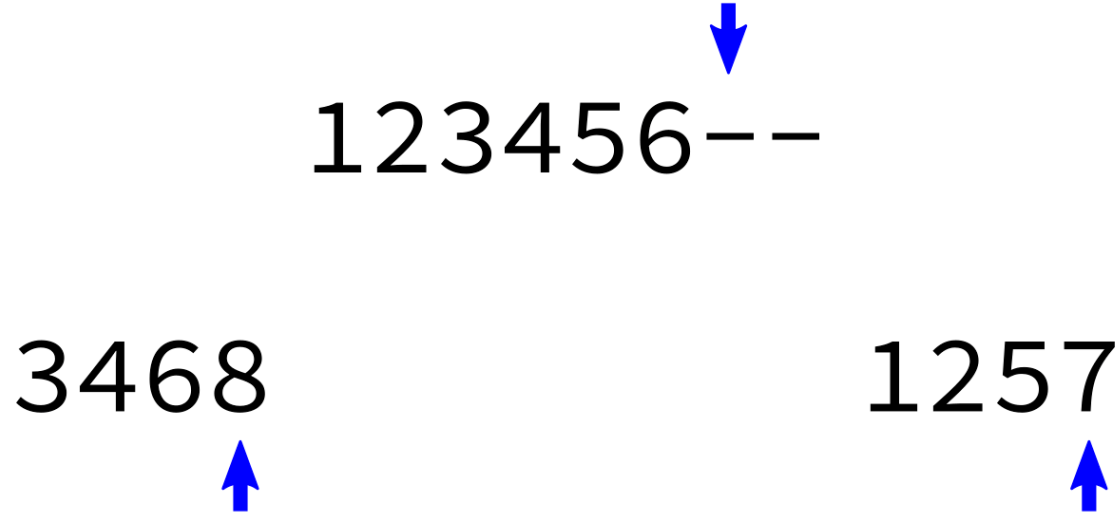

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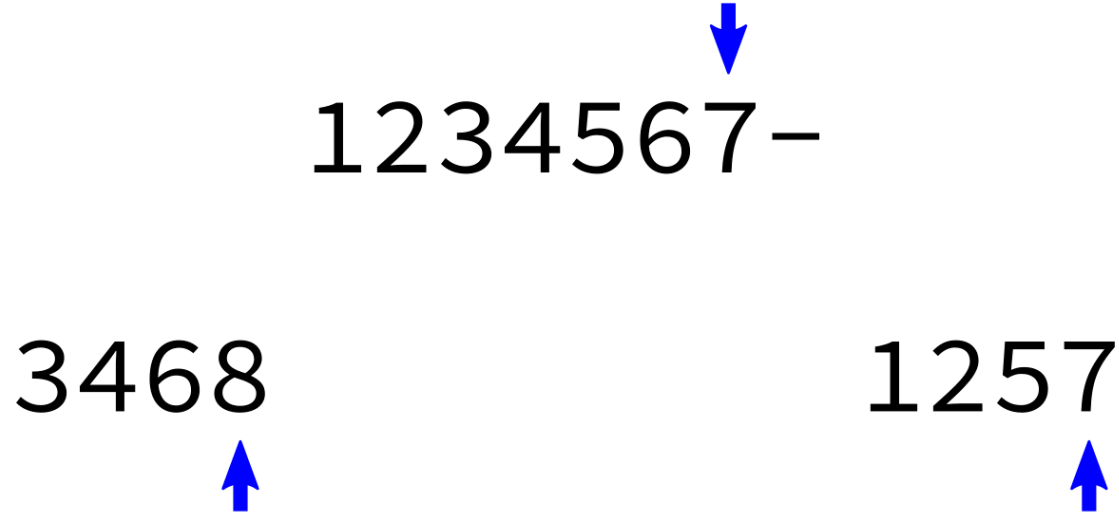

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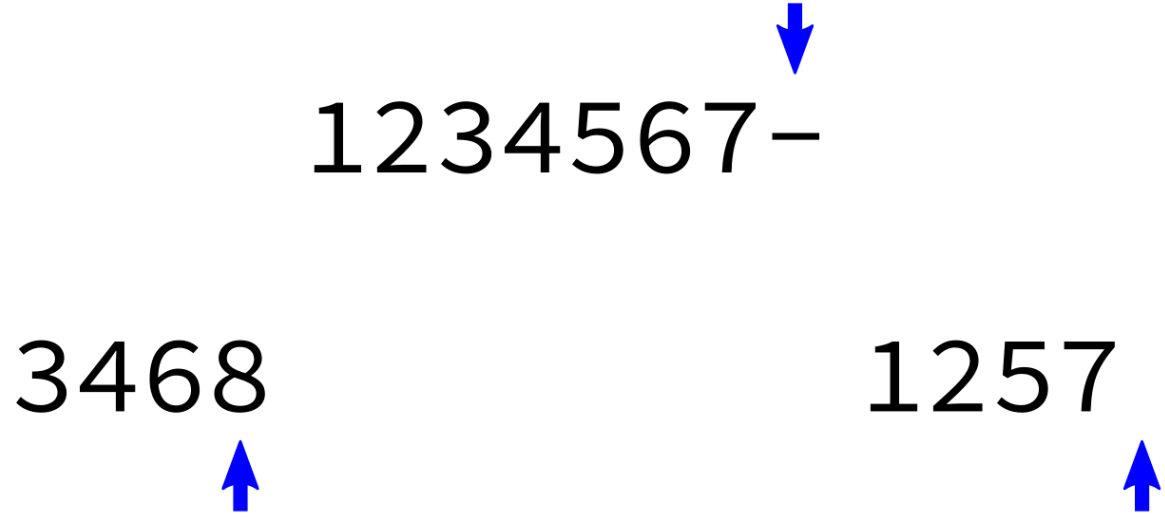

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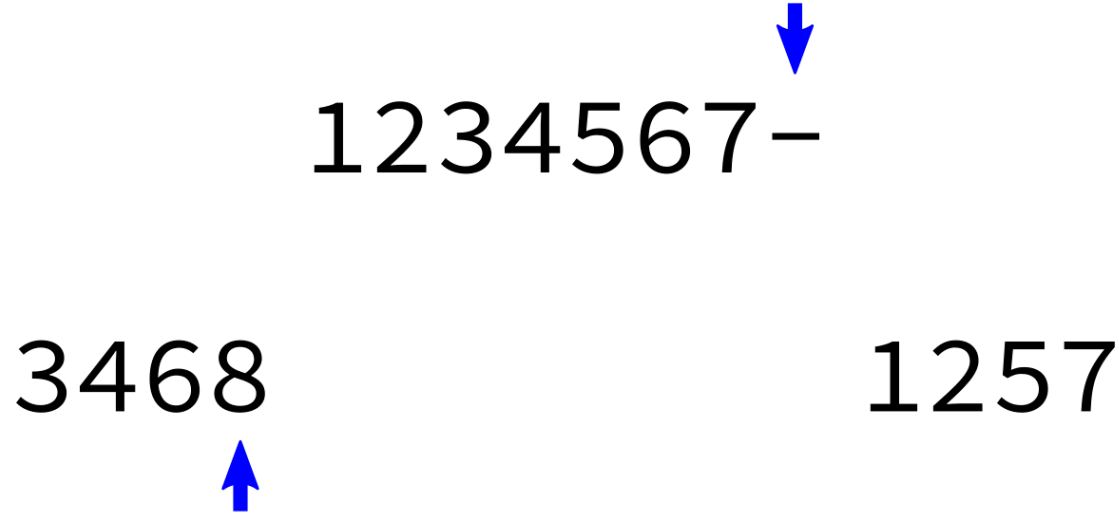

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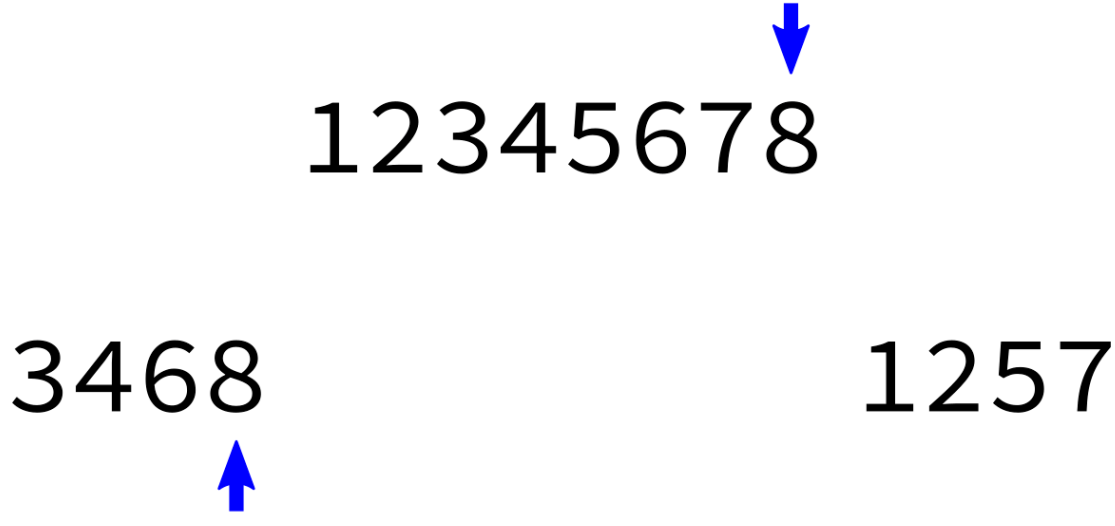

MERGING SORTED LISTS

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MERGING SORTED LISTS

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CODING TIME

Let's implement `mergesort` in Python.

REFERENCES

No changes to the references from Lecture 13

- Algorithms by Jeff Erickson, Chapter 1. Mergesort is example 1.4.
- Lutz discusses recursive functions in Chapter 19 (pages 555-559 in the print edition).
- Intro to Python for Computer Science and Data Science by Deitel and Deitel, Chapter 11.
- Think Python, 2ed, by Allen B. Downey, Sections 5.8 to 5.10.
- Computer Science: An Overview by Brookshear and Brylow, Section 5.5.

REVISION HISTORY

- 2021-02-18 Move unused slides to Lecture 17
- 2021-02-17 Initial publication