# LECTURE 14

## RECURSION VS ITERATION II

MCS 275 Spring 2021
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# LECTURE 14: RECURSION VS ITERATION II

Course bulletins:

- Project 2 description available.
- Project 2 due 6pm CST Friday, February 26.
- Check out the recursion sample code.

# PLAN

- A bit about project 2
- More on recursion, iteration, counting function calls
- Start on backtracking

# PROJECT 2 TOPIC

Focuses on recursion. Based on special classes of strings that have the "pattern" `ABB`:

- **Egg:** `A` and `B` are single characters, e.g. `egg`, `off`, `aaa`
- **Superegg:** `A` is a superegg or single character and `B` is a single character, e.g. `add`, `addee`
- **Hyperegg:** `A` and `B` are each hypereggs or single characters, e.g. `anoonoo`, `offeggegg`, `gooss`

# PROJECT 2 TASK

You'll write functions to test whether a string belongs to these classes.

Details in the project description.

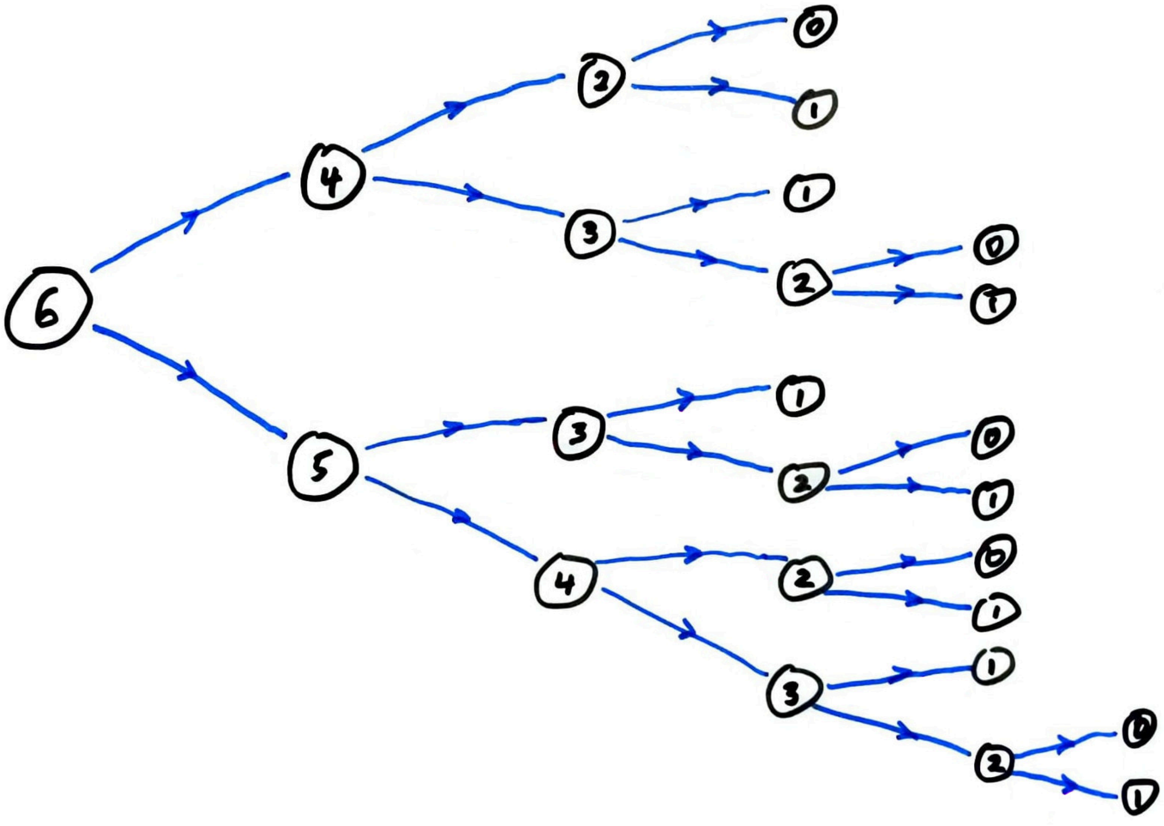
Later I will provide test data, but you'll need to write your own test code.

# FIBONACCI TIMING

| | **n=35** |
|---|---|
| recursive | 1.9s |
| iterative | <0.001s |

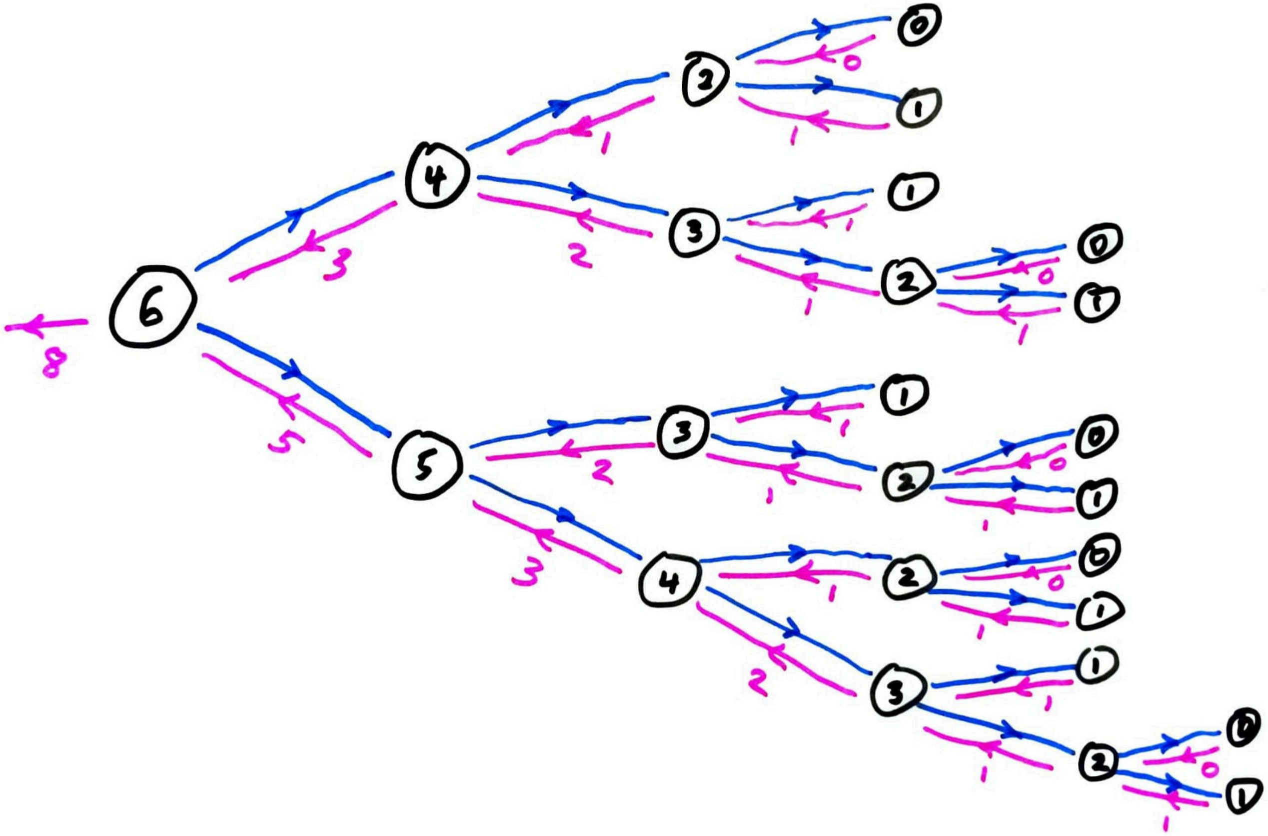
Measured on a 4.00Ghz Intel i7-6700K CPU (2015 release date) with Python 3.8.5

# FIB CALL GRAPH



Most Fibonacci numbers are computed many times!

# FIB CALL GRAPH



Most Fibonacci numbers are computed many times!

# MEMOIZATION

`fib` computes the same terms over and over again.

Instead, let's store all previously computed results, and use the stored ones whenever possible.

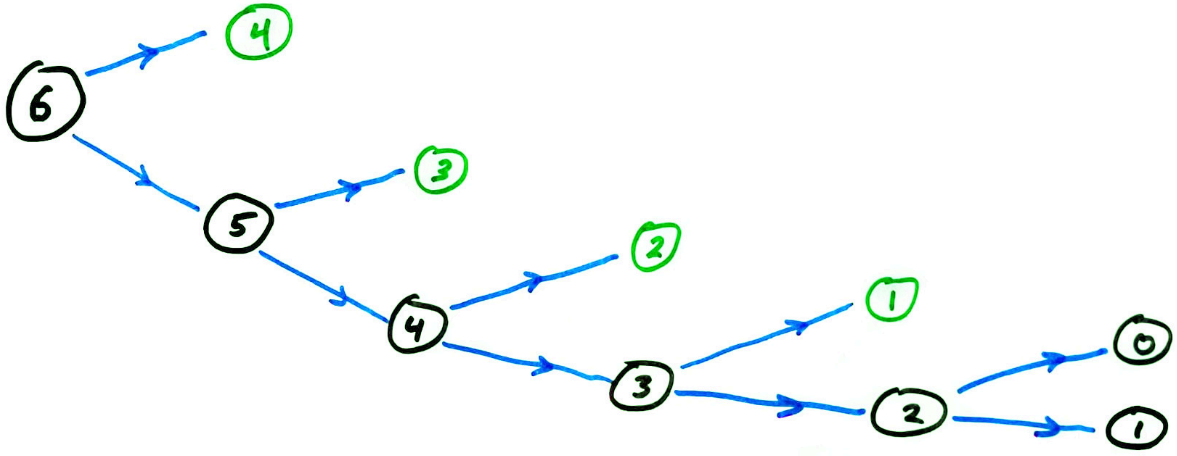
This is called **memoization**. It only works for **pure functions**, i.e. those which always produce the same return value for any given argument values.

`math.sin(...)` is pure; `time.time()` is not.

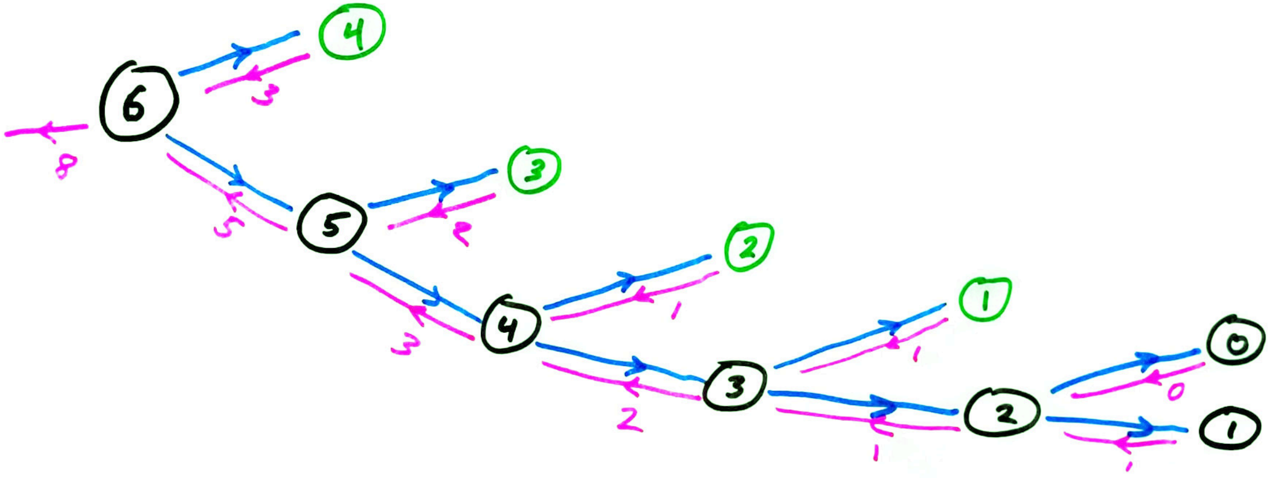
# MEMOIZING FIB

Let's add a simple memoization feature to our recursive `fib` function.

# MEMOIZED FIB CALL GRAPH



# MEMOIZED FIB CALL GRAPH



# FIBONACCI TIMING SUMMARY

| | **n=35** | **n=450** |
|---|---|---|
| recursive | 1.9s | > age of universe |
| memoized recursive | <0.001s | 0.003s |
| iterative | <0.001s | 0.001s |

Measured on a 4.00Ghz Intel i7-6700K CPU (2015 release date) with Python 3.8.5

# MEMOIZATION SUMMARY

Recursive functions with multiple self-calls often benefit from memoization.

Memoized version is conceptually similar to an iterative solution.

Memoization does not alleviate recursion depth limits.

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | | | | | | | |

# CALL COUNTS

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Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | 1 | | | | | | |

# CALL COUNTS

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| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | `1` | `1` | | | | | |

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | 1 | 1 | 3 | | | | |

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | 1 | 1 | 3 | 5 | | | |

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | 1 | 1 | 3 | 5 | 9 |  |  |

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | 1 | 1 | 3 | 5 | 9 | 15 | |

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|---|
| **calls** | 1 | 1 | 3 | 5 | 9 | 15 | 25 |

# CALL COUNTS

One way to measure the expense of a recursive function is to count how many times the function is called.

Let's do this for recursive `fib`.

| $n$ | **0** | **1** | **2** | **3** | **4** | **5** | **6** | |
|---|---|---|---|---|---|---|---|---|
| **calls** | 1 | 1 | 3 | 5 | 9 | 15 | 25 | |
| $F_n$ | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 |

**Theorem:** Let $T(n)$ denote the total number of times `fib` is called to compute `fib(n)`. Then

$$T(0) = T(1) = 1$$

and

$$T(n) = T(n-1) + T(n-2) + 1.$$

**Corollary:** $T(n) = 2F_{n+1} - 1$.

**Proof of corollary:** Let $S(n) = 2F_{n+1} - 1$. Then $S(0) = S(1) = 1$, and

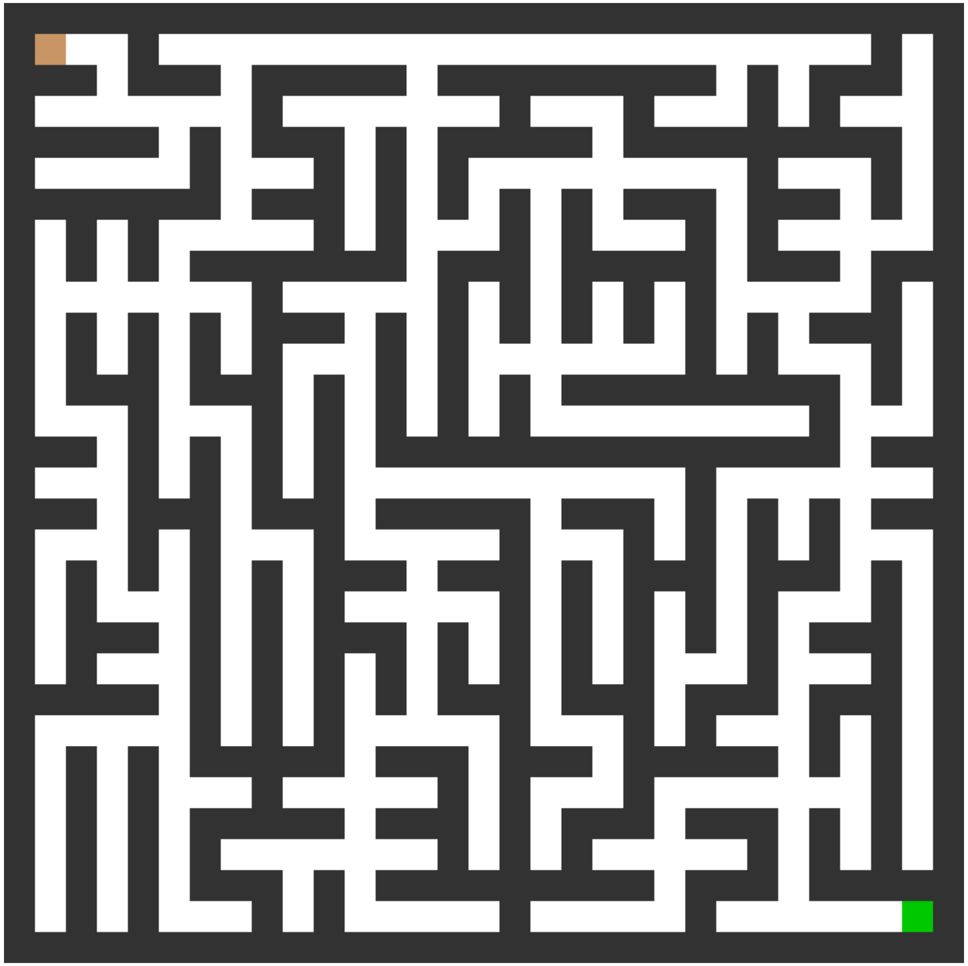
$$\begin{aligned} S(n) &= 2F_{n+1} - 1 = 2(F_n + F_{n-1}) - 1 \\ &= (2F_n - 1) + (2F_{n-1} - 1) + 1 \\ &= S(n-1) + S(n-2) + 1 \end{aligned}$$

Therefore $S$ and $T$ have the same first two terms, and follow the same recursive definition based on the two

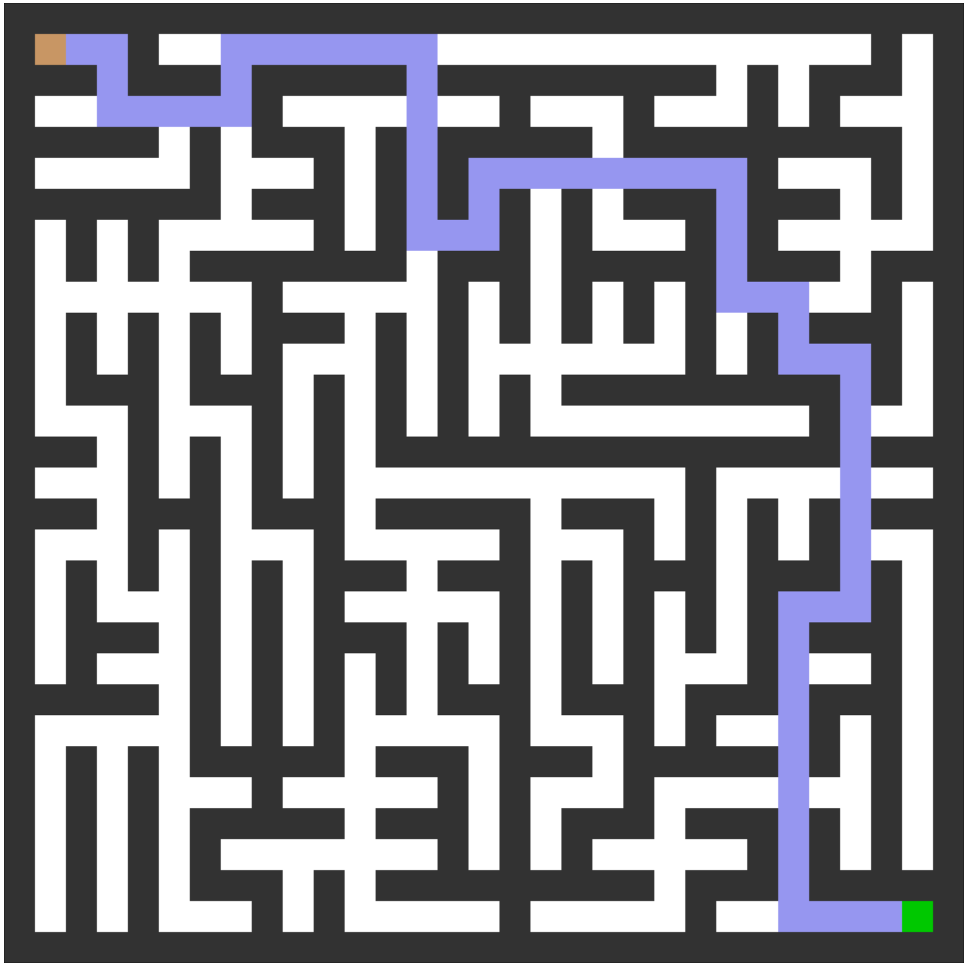
**Corollary:** Every time we increase $n$ by 1, the naive recursive `fib` does $\approx 61.8\%$ more work.

(The ratio $F_{n+1}/F_n$ approaches $\frac{1+\sqrt{5}}{2} \approx 1.61803$.)

# RECURSION WITH BACKTRACKING



# RECURSION WITH BACKTRACKING



# REFERENCES

No changes to the references from Lecture 13

- Algorithms by Jeff Erickson, Chapter 1.
- Lutz discusses recursive functions in Chapter 19 (pages 555-559 in the print edition).
- Intro to Python for Computer Science and Data Science by Deitel and Deitel, Chapter 11.
- Think Python, 2ed, by Allen B. Downey, Sections 5.8 to 5.10.
- Computer Science: An Overview by Brookshear and Brylow, Section 5.5.

# REVISION HISTORY

- 2021-02-15 Move some unused slides over to Lecture 15
- 2021-02-12 Initial publication