LECTURE 13

RECURSION VS ITERATION

MCS 275 Spring 2021
Emily Dumas

LECTURE 13: RECURSION VS ITERATION

Course bulletins:

- Please complete anonymous feedback survey (link in Blackboard announcement)
- Project 1 grades and solutions posted
- Project 2 will be posted by Friday; due Feb 26

LOOSE END

Let's implement the paper folding sequence recursively.

COOL FACT

If you use the infinite paper folding sequence as the binary digits of a real number (starting at $\frac{1}{2}$ place and moving right), you get the **paper folding constant**.

$$\begin{aligned} PFC &= (0.1101100111001001110110 \ldots)_2 \\ &= 0.85073618820186 \ldots \end{aligned}$$

It is irrational. In 2007 it was shown[1] that this constant is furthermore **transcendental**, i.e. cannot be expressed in terms of square roots, cube roots, or any solutions of polynomials with rational coefficients.

1 Adamczewski and Bugeaud, *On the complexity of algebraic numbers I: Exapansions in integer bases*, Annals of Mathematics 165 (2007) 547-565.

STACK OVERFLOW

Recursive functions are limited by a maximum call stack size.

Python imposes a limit to prevent the memory area used to store the call stack from running out (a stack overflow), which would abruptly stop the interpreter.

ITERATIVE SOLUTIONS

Let's write iterative versions of factorial, Fibonacci, and paper folding. (Or as many as time allows.)

TIMING COMPARISON

How do iterative and recursive versions compare on speed?

I made a module `decs` contains a decorator called `timed` that prints the time a function takes to return.

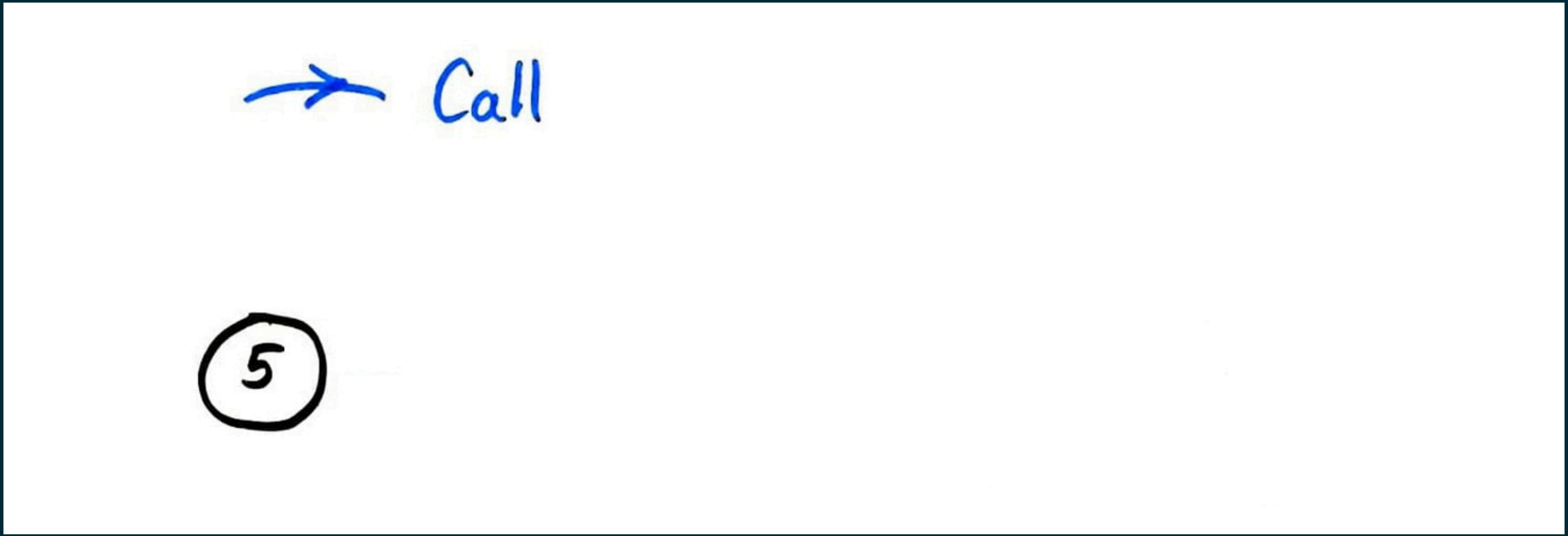
QUESTION

Why is recursive `fact()` somewhat competitive, but `fib()` is dreadfully slow?

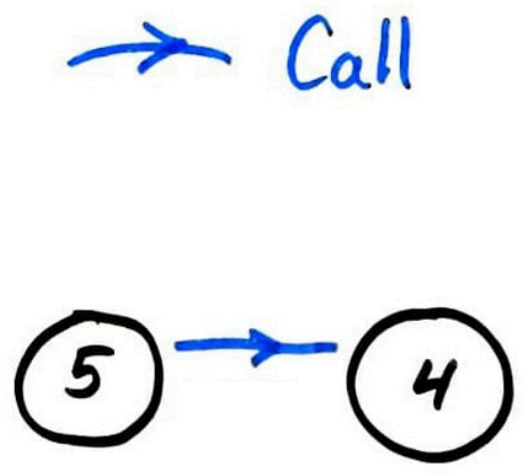
Decorator `decs.count_calls` will keep track of number of function calls.

FACT CALL GRAPH

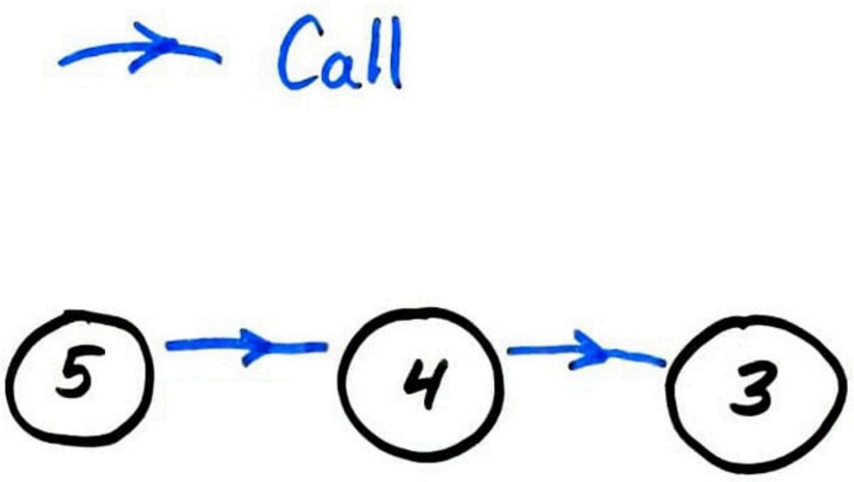
FACT CALL GRAPH



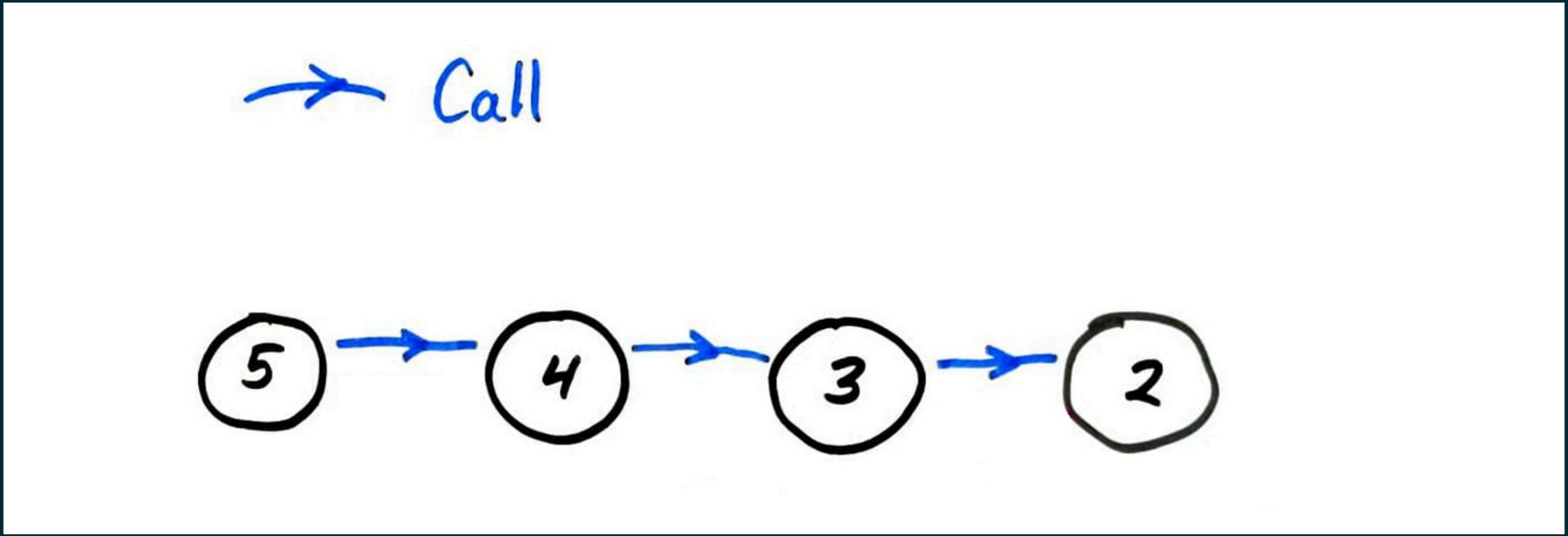
FACT CALL GRAPH



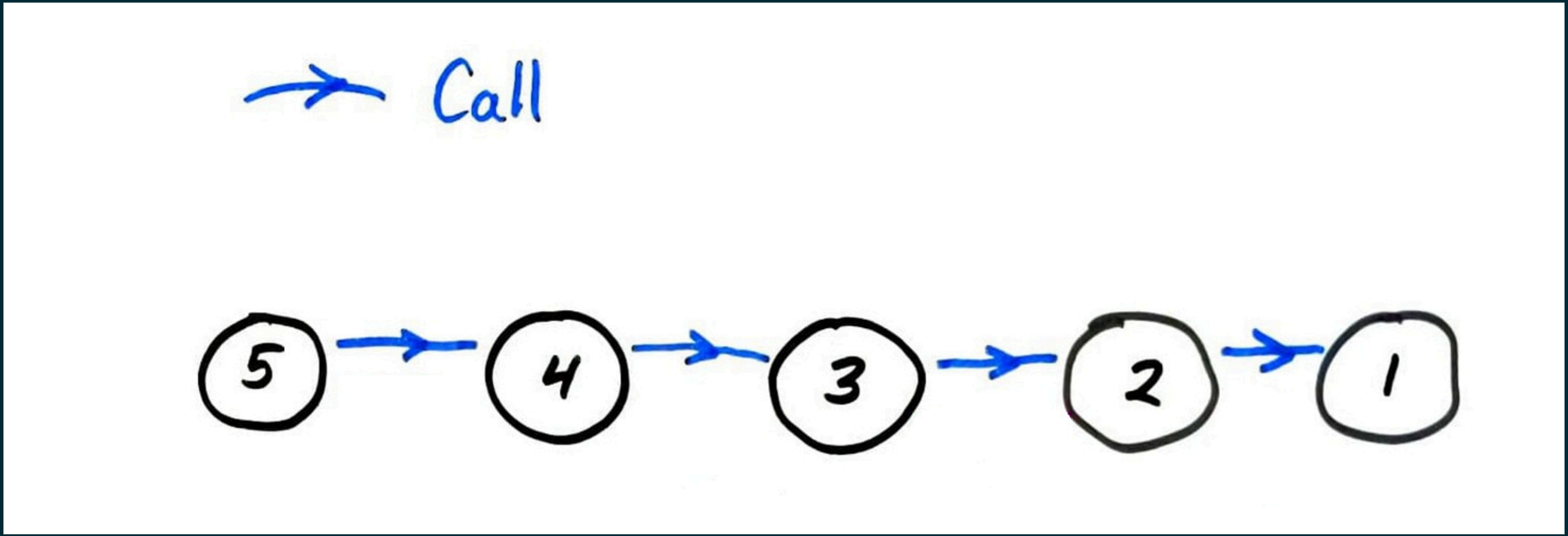
FACT CALL GRAPH


$$\begin{tikzcd} {} \arrow[r] & \text{Call} \end{tikzcd}$$
$$\begin{tikzcd} 5 \arrow[r] & 4 \arrow[r] & 3 \end{tikzcd}$$

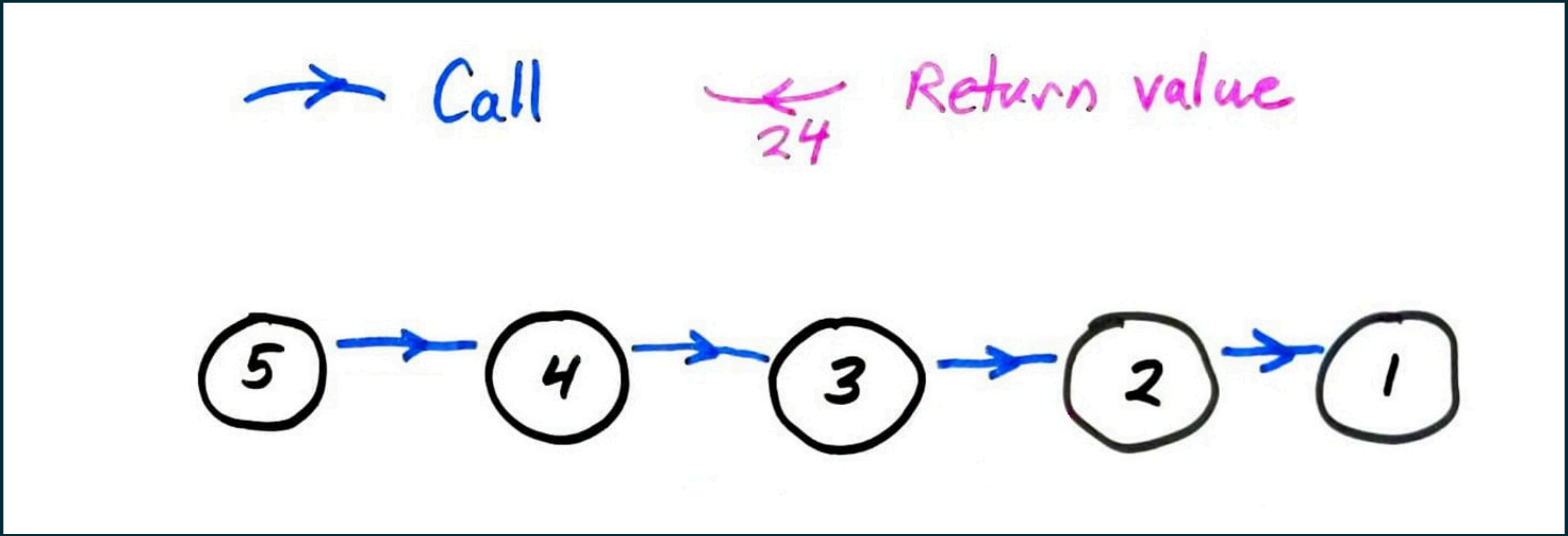
FACT CALL GRAPH



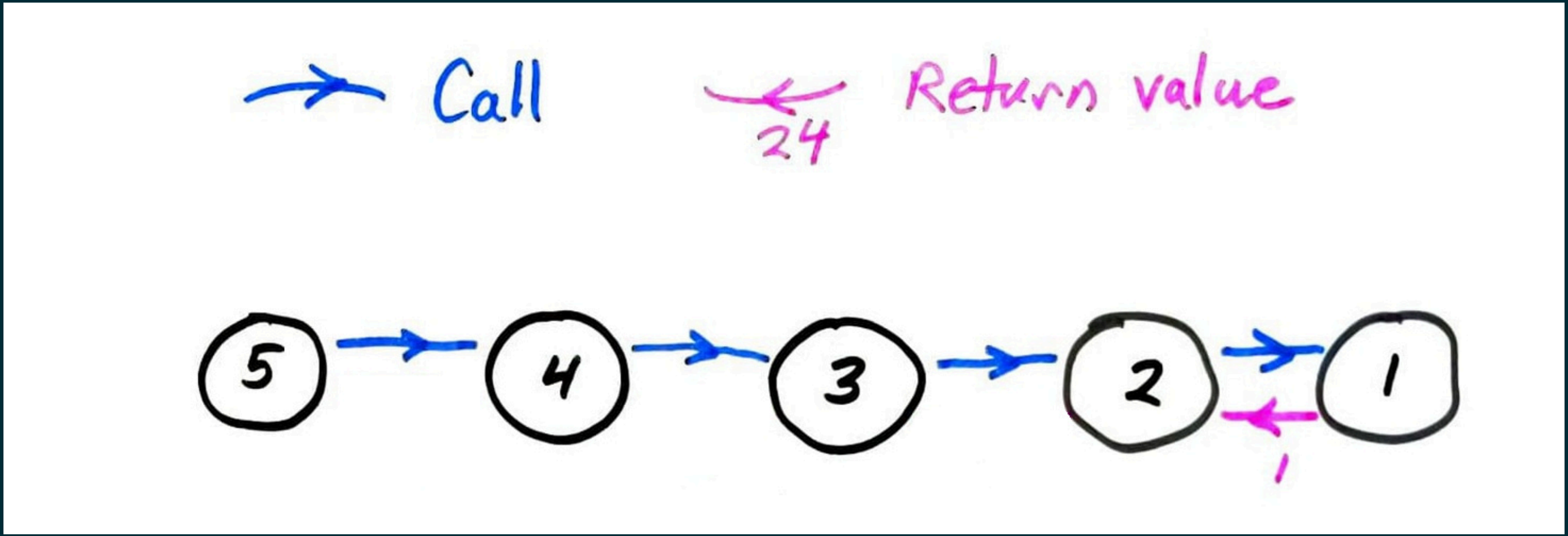
FACT CALL GRAPH



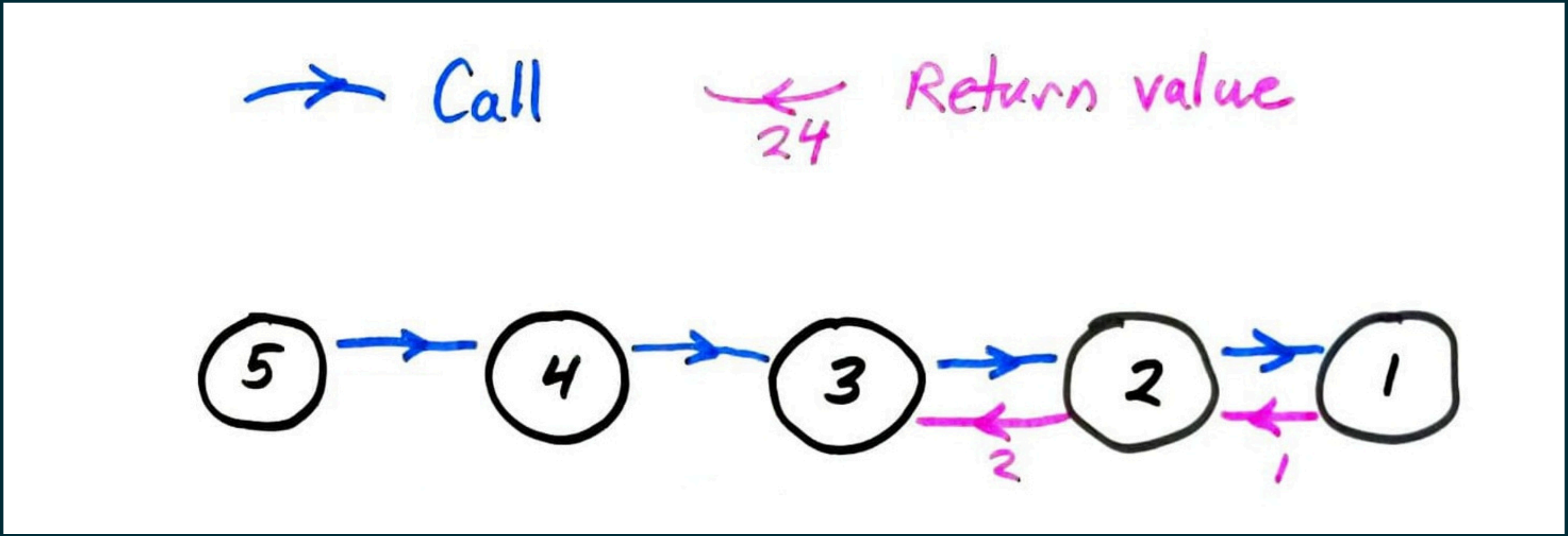
FACT CALL GRAPH



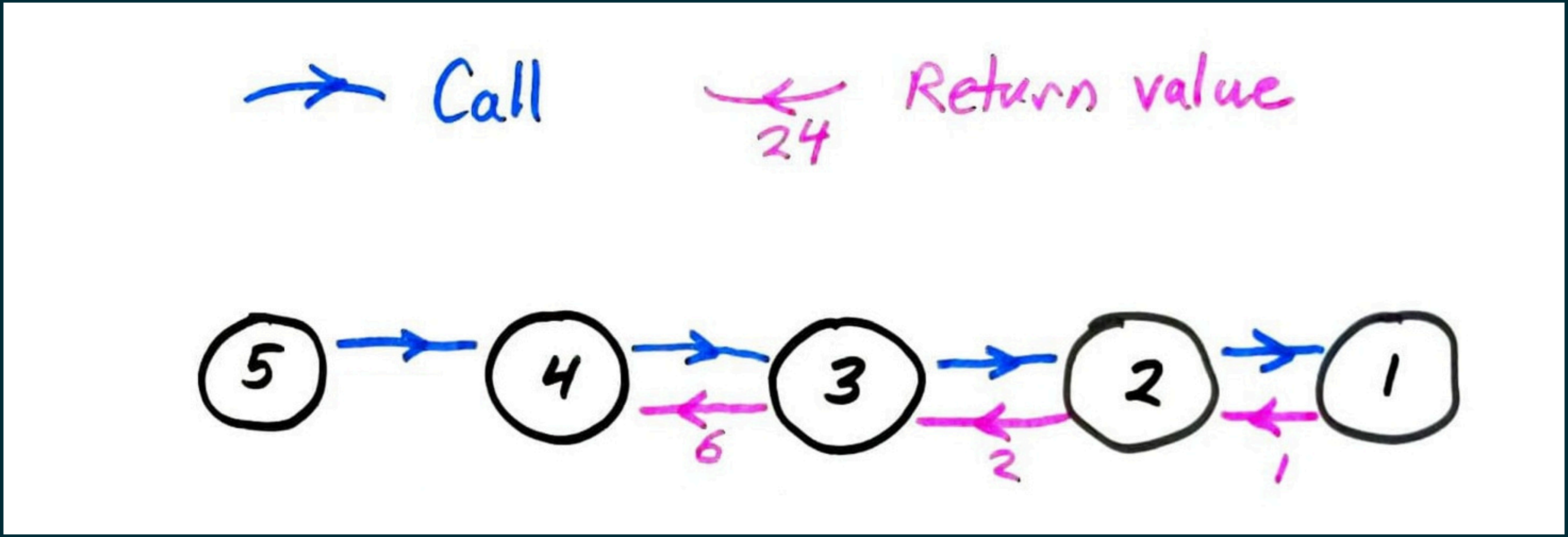
FACT CALL GRAPH



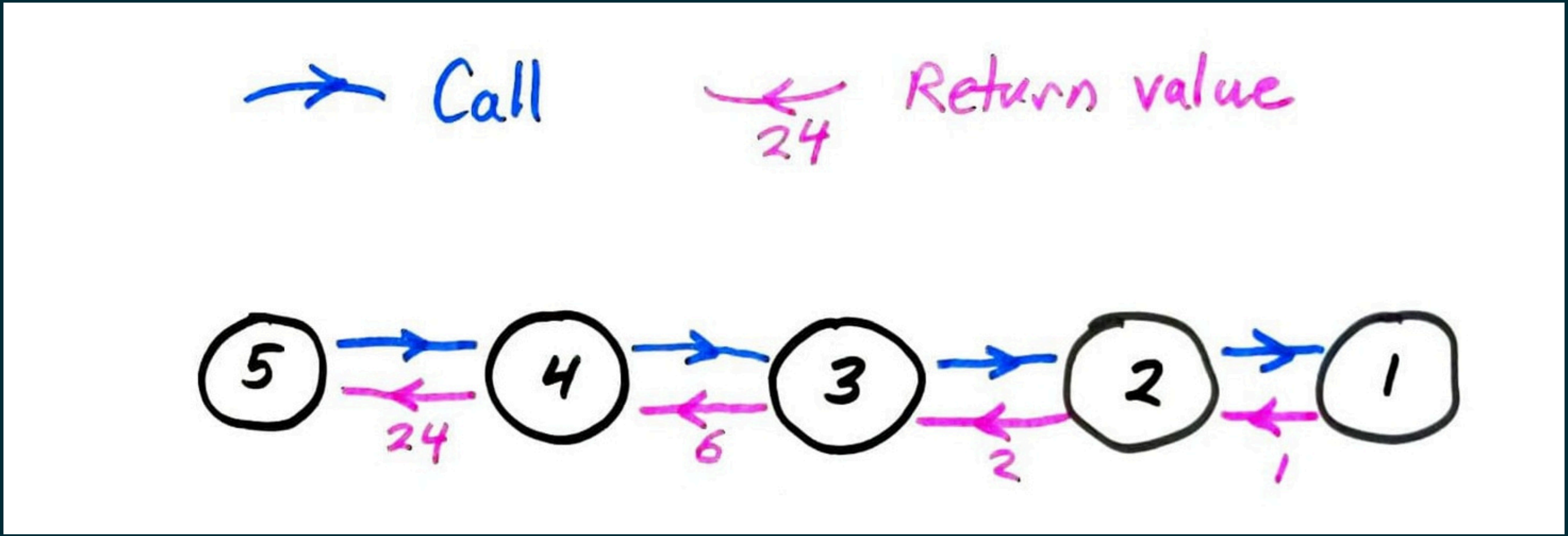
FACT CALL GRAPH



FACT CALL GRAPH

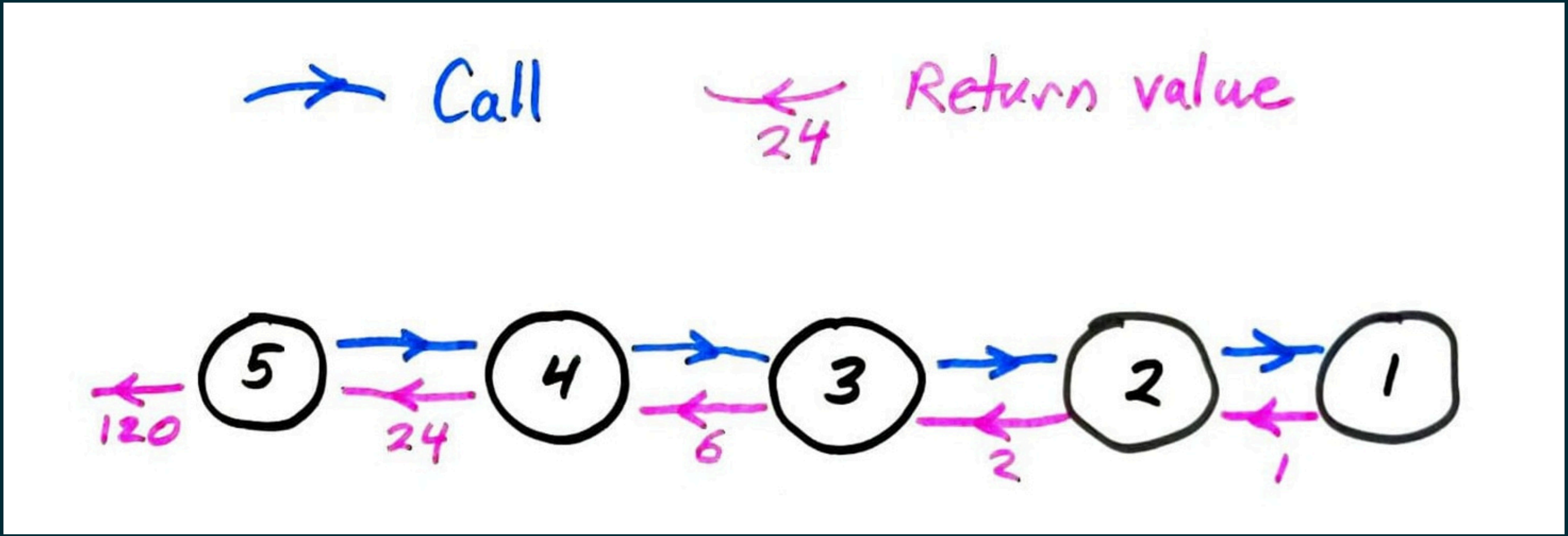


FACT CALL GRAPH

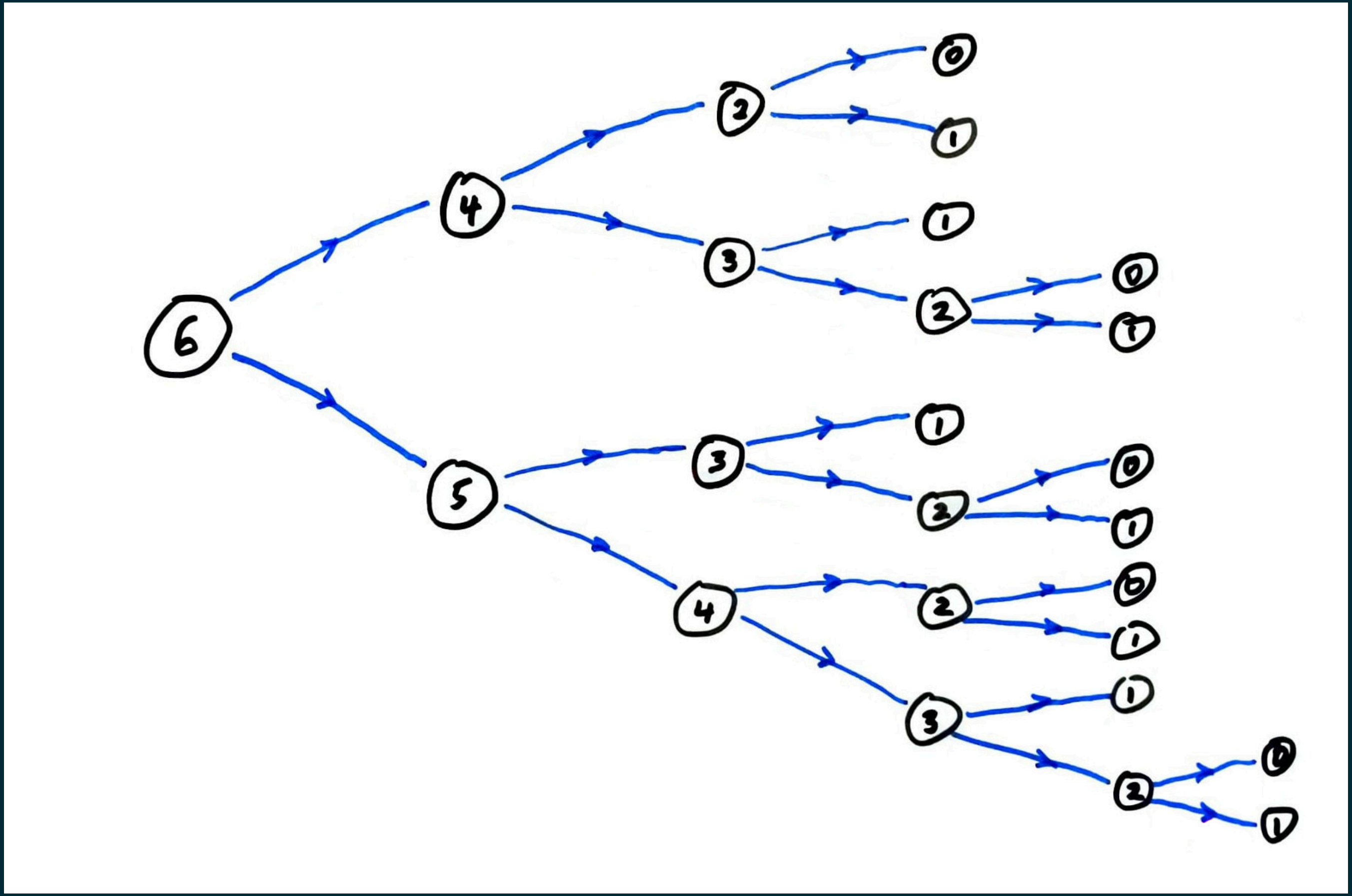


$$\begin{tikzcd}
5 \arrow[r, shift left] & 4 \arrow[l, shift left, "24"] \arrow[r, shift left] & 3 \arrow[l, shift left, "6"] \arrow[r, shift left] & 2 \arrow[l, shift left, "2"] \arrow[r, shift left] & 1 \arrow[l, shift left, "1"]
\end{tikzcd}$$

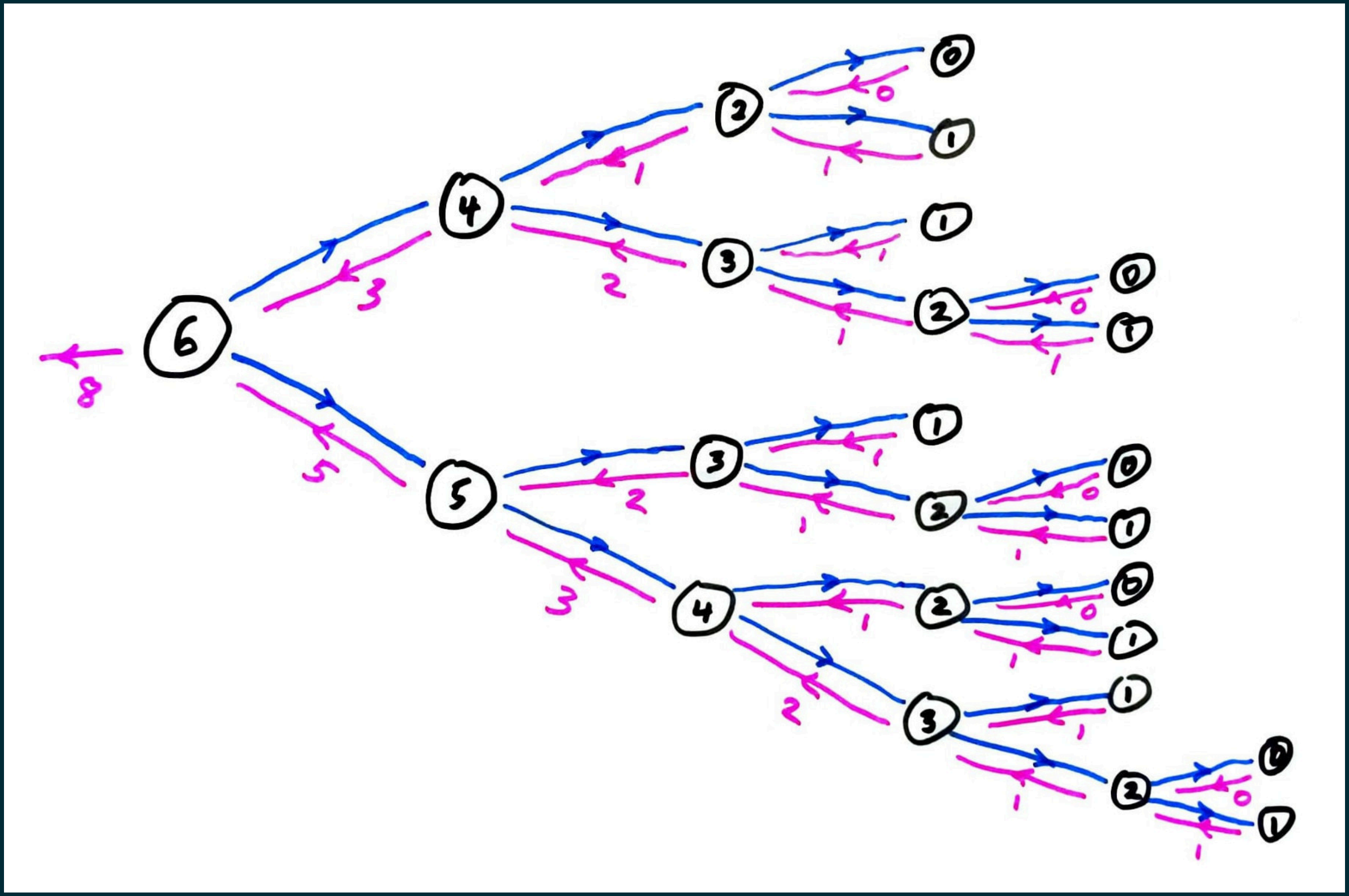
FACT CALL GRAPH



FIB CALL GRAPH



FIB CALL GRAPH



MEMOIZATION

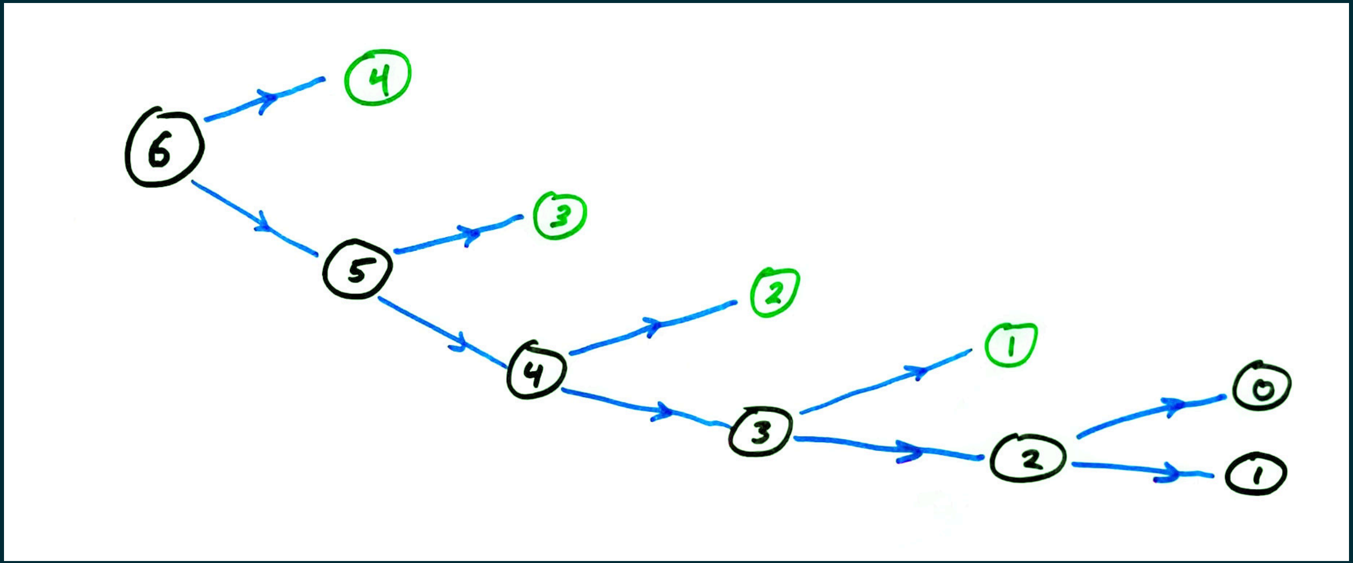
`fib` computes the same terms over and over again.

Instead, let's store all previously computed results, and use the stored ones whenever possible.

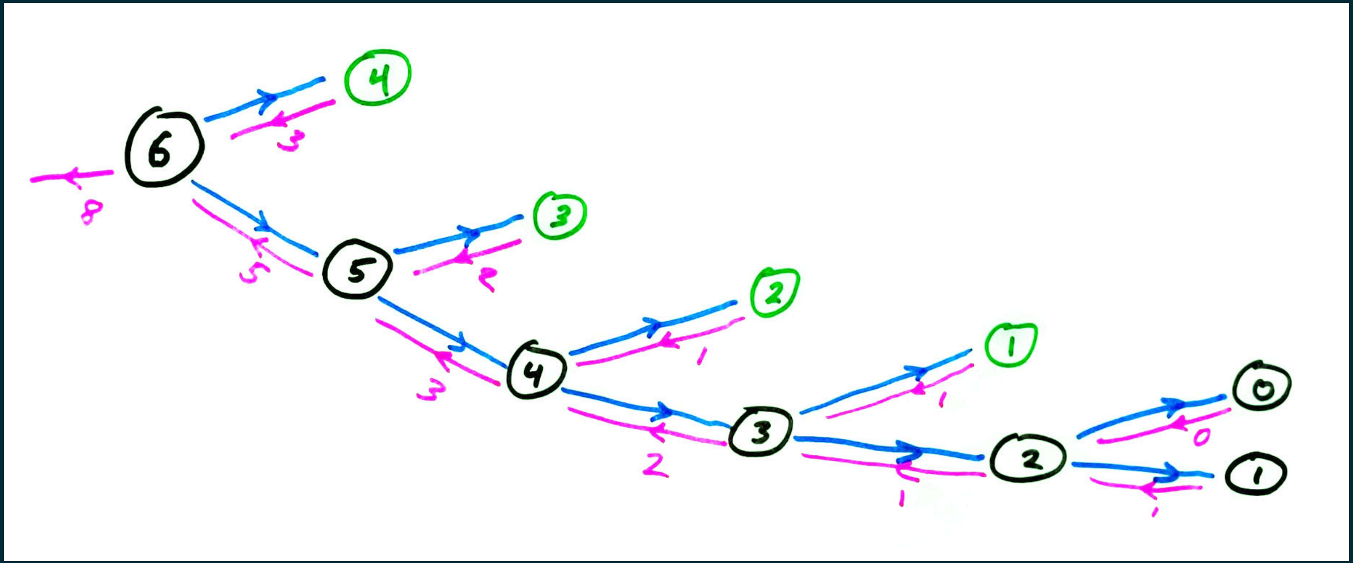
This is called **memoization**. It only works for **pure functions**, i.e. those which always produce the same return value for any given argument values.

`math.sin(...)` is pure; `time.time()` is not.

MEMOIZED FIB CALL GRAPH



MEMOIZED FIB CALL GRAPH



FIBONACCI TIMING SUMMARY

	n=35	n=450
recursive	1.9s	> age of universe
memoized recursive	<0.001s	0.003s
iterative	<0.001s	0.001s

Measured on a 4.00Ghz Intel i7-6700K CPU (2015 release date) with Python 3.8.5

MEMOIZATION SUMMARY

Recursive functions with multiple self-calls often benefit from memoization.

Memoized version is conceptually similar to an iterative solution.

Memoization does not alleviate recursion depth limits.

REFERENCES

- *Algorithms* by Jeff Erickson, available as a free PDF, discusses some examples of recursion in Chapter 1.
- *Lutz* discusses recursive functions in Chapter 19 (pages 555-559 in the print edition).
- *Intro to Python for Computer Science and Data Science* by Deitel and Deitel discusses recursion in Chapter 11. The online version of this text is freely available to UIC students, faculty, and staff. (You will first need to log in with you UIC email.)
- The open textbook *Think Python*, 2ed, by Allen B. Downey discusses recursion in Sections 5.8 to 5.10.
- *Computer Science: An Overview* by Brookshear and Brylow discusses recursion in Section 5.5. (This book is often an optional text for MCS 260.)

REVISION HISTORY

- 2021-02-10 Add reference for transcendence of PFC
- 2021-02-10 Initial publication