

Homework 9

Due Wednesday, April 3 in class (1:00pm)

(—) From the textbook: 27.2abc, 27.4*, 27.6ab

* Hint: This problem does not involve compactness! However, some of the constructions in metric spaces that are used in section 27 may be useful in devising a solution.

(P1) Determine whether or not $[0, 1]$ is compact in the lower limit topology.

(P2) Determine whether or not $\mathbb{Q} \cap [0, 1]$ is compact in the standard topology.

(P3) Let X be a Hausdorff topological space.

(a) If A_1, A_2 are compact subsets of X , show that $A_1 \cap A_2$ is compact.

(b) If $\{A_\alpha\}_{\alpha \in J}$ is a collection of compact subsets of X , it is always the case that $\bigcap_{\alpha \in J} A_\alpha$ is compact?

(As usual, you need to prove everything: If the answer is yes, prove that the intersection is compact under the given hypotheses. If the answer is no, give a counterexample, and prove that your example has the necessary properties.)