Math 445 – David Dumas – Spring 2019

Homework 5

Due Monday, February 25 in class (1:00pm)

(—) From the textbook: 20.4a*, 20.5, 20.6, 21.2

$*$ Note this is actually nine separate questions: For each of three topologies, and for each of three functions f, g, h, you need to indicate whether it is continuous and give a proof. Please organize your answer so that f, g, and h are discussed separately. Use relations between the given topologies (finer/coarser) to make your proofs as concise as possible. Finally, please include in your answer a table giving all of the yes/no continuity answers in the following format:

	prod	unif	box
f	?	?	?
g	?	?	?
h	?	?	?

(P1) Let (X,d) be a metric space.

(a) Define $d_2 : X \times X \to \mathbb{R}$ by $d_2(x,y) = (d(x,y))^2$. Is d_2 a metric on X? Give a proof or a counterexample.

(b) Define $d_{\frac{1}{2}} : X \times X \to \mathbb{R}$ by $d_{\frac{1}{2}}(x,y) = \sqrt{d(x,y)}$. Is $d_{\frac{1}{2}}$ a metric on X? Give a proof or a counterexample.

(c) If the answer to any of the previous parts was "yes", determine whether the metric topology associated to this new metric is coarser, finer, or equal to the one from d.

(P2) Suppose (X,d) is a metric space and $f : X \to X$ is a function such that $d(f(x), f(y)) \leq d(x,y)$ for all $x, y \in X$. Show that f is continuous.