Math 445 – David Dumas – Spring 2019

Final Exam Problems 1 & 2

Instructions:

- The final exam will have **six** problems, and you will be asked to complete any **four** of them.
- The first two problems from the final exam are shown below. You may work on these problems in preparation for the exam. Keep in mind that you will not be able to bring any notes to the exam.
- Collaboration (or receiving assistance of any kind) on these problems is prohibited.

(1) Let X be a Hausdorff topological space and $A \subset X$ a dense subset. If the subspace topology on A is locally compact, show that A is open in X.

(2) Let (X,d) be a metric space, and let A and B be disjoint closed subsets of X. For any $x \in X$ let

$$f(x) = \frac{d(x,A)}{d(x,A) + d(x,B)}.$$

Show that this gives a continous function $f : X \to \mathbb{R}$ with $f(A) = \{0\}$ and $f(B) = \{1\}$.