

Midterm Exam

Note: On this exam we use the notation $a \times b$ for an element of a Cartesian product $A \times B$.

Instructions: Complete any three problems.

1. Consider the ordered set $X = \mathbb{N} \times \mathbb{N}$ with the dictionary order. In the order topology on X , determine whether each of the following sequences converges. If the sequence does converge, identify the point it converges to. In each case give a proof.
 - (a) $a_n = n \times 2$
 - (b) $b_n = 2 \times n$
2.
 - (a) Write the definition of a *Hausdorff* topological space.
 - (b) Prove that if X is a metrizable topological space, then X is Hausdorff.
3.
 - (a) Write the definition of the *lower limit topology* on \mathbb{R} .
 - (b) Let $A \subset \mathbb{R}$ denote the set of rational numbers in the interval $(0, 1)$. Determine the closure of A in the lower limit topology on \mathbb{R} .
4. Let X be a topological space.
 - (a) Write the definition of a *limit point* of a set $A \subset X$.
 - (b) Suppose $A, B \subset X$ and that x is a limit point of $A \cup B$. Show that either x is a limit point of A or x is a limit point of B (or both).
5.
 - (a) Let X be a topological space and \sim an equivalence relation on X . Write the definition of the *quotient topology* on X/\sim .
 - (b) Let $X = A \times B$ where A and B are topological spaces. Define an equivalence relation on X as follows:

$$a_1 \times b_1 \sim a_2 \times b_2 \text{ if and only if } a_1 = a_2$$

Show that X/\sim with the quotient topology is homeomorphic to A .