Math 445 – David Dumas – Spring 2018

Homework 9

Due Monday, April 2 in class (1:00pm)

Follow the same instructions given on Homework 1.

(—) From the textbook: 26.5, 26.12, 27.2, 28.6

(P1) Let X be a compact metric space. Show that there is a countable subset $N \subset X$ that is *dense*, i.e. so that $X = \bar{N}$. (Hint: Finitely cover X by smaller and smaller balls.)

(P2) Show that the only compact, connected subsets of $\mathbb{R}$ are the closed intervals.

(P3) Consider $\mathbb{R}^\omega$ with the uniform topology. (Recall this is the topology of the metric $\bar{\rho}$ described in §20 of the text.) Define

$$B = \{x \in \mathbb{R}^\omega \mid \bar{\rho}(\vec{0}, x) \leq \frac{1}{2}\}.$$

Is B compact?