Math 445 – David Dumas – Spring 2018

Homework 6

Due Monday, March 5 in class (1:00pm)

Follow the same instructions given on Homework 1.

(—) From the textbook: 21.7, 22.2, 22.4

(P1) Recall that in Theorem 20.5, our textbook shows that the function $D : \mathbb{R}^\omega \times \mathbb{R}^\omega \to \mathbb{R}$ defined by

$$D(x,y) = \sup \left\{ \frac{\bar{d}(x_i,y_i)}{i} \;\middle|\; i \in \mathbb{N} \right\}$$

is a metric, where $\bar{d}(s,t) = \min\{|s-t|,1\}$.

Consider the function

$$D'(x,y) = \sup \left\{ \frac{\bar{d}(x_i,y_i)}{2^i} \;\middle|\; i \in \mathbb{N} \right\}.$$

(a) Show that D' is a metric on $\mathbb{R}^\omega$.

(b) Compare the topologies induced by D and D': Is one finer than the other? Are they the same? (Give a proof.)

(P2) Let $A = \prod_{i\in\mathbb{N}}(0,1)$ denote the product of countably many copies of the interval $(0,1)$ in $\mathbb{R}$. We can give A the subspace topology for any of the three topologies on $\mathbb{R}^\omega$ we have studied: The product topology, the uniform topology, or the box topology.

Consider the function $S : A \to \mathbb{R}$ defined by

$$S(x) = \sup\{x_i \mid i \in \mathbb{N}\}.$$

For each of the three topologies on A described above, determine whether or not S is a continuous function.