Math 445 – David Dumas – Spring 2018

Homework 5

Due Monday, February 26 in class (1:00pm)

Follow the same instructions given on Homework 1.

(—) From the textbook: 20.2, 20.3, 20.4, 20.6, 21.2

(P1) Let $f : \mathbb{R} \to \mathbb{R}^\omega$ be a function. Let $f_i = \pi_i \circ f : \mathbb{R} \to \mathbb{R}$ denote the i^{th} coordinate function of f. Suppose that for all i the function f_i is continuous, and furthermore that all but finitely many of these functions are constant. Show that f is continuous with respect to the box topology on $\mathbb{R}^\omega$.

This problem was corrected on February 20.

(P2) Does there exist a function $f : \mathbb{R} \to \mathbb{R}^\omega$ that is continuous with respect to the box topology and in which none of the coordinate functions f_i are constant functions? Either construct one (including a proof that the function you construct is continuous) or show that no such function exists.

(P3) Let $C([0,1])$ denote the set of real-valued functions on the interval $[0,1]$ that are continuous with respect to the standard topology. Given elements $f,g \in C([0,1])$, define

$$d_\infty(f,g) = \sup_{x\in[0,1]} |f(x) - g(x)|.$$

and

$$d_1(f,g) = \int_0^1 |f(x) - g(x)|dx.$$

(a) Show that d_∞ is a metric on $C([0,1])$.

(b) Show that d_1 is a metric on $C([0,1])$.

(c) Do d_∞ and d_1 define the same topology on $C([0,1])$?