

Homework 12

Due Monday, April 23 in class (1:00pm)

Follow the same instructions given on [Homework 1](#).

(—) From the textbook: 37.1, 37.2, 43.1, 43.4

(P1) Let \mathcal{B}_1 denote the collection of all open intervals in \mathbb{R} . Define

$$\mathcal{B}_2 = \{B \cap \mathbb{Q} \mid B \in \mathcal{B}_1\}.$$

Let $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$.

(a) Show that \mathcal{B} is a basis on \mathbb{R} .

In the rest of the problem, let X denote the topological space that is the set \mathbb{R} with the topology $\mathcal{T}(\mathcal{B})$ generated by \mathcal{B} .

(a) Show that X is second countable.

(b) Show that X is Hausdorff.

(c) Show that X is not regular, and hence not metrizable.

(Hint: Suppose it is regular, and let $a = 0$ and $B = \mathbb{R} - \mathbb{Q}$. Derive a contradiction.)

(P2) Give an example of a compact Hausdorff space that is not metrizable.

(Hint: Asking this after covering §37 makes a relatively simple example accessible.)

Note: The textbook problem list on this assignment was corrected on April 15.