Math 445 – David Dumas – Spring 2018

Homework 1

Due Monday, January 29 in class (1:00pm)

Notes and instructions:

- Problems from the textbook (Munkres, 2ed) are specified in the format "Section.Exercise". For example, "13.8" means exercise 8 from section 13, which is the last problem on page 83.
- When writing your solutions, clearly label each one with the problem number. Use the same conventions as in the homework sheet (i.e. 13.8, 14.2, etc., for book problems, P1, P2, etc. for non-book problems).
- To receive full credit, a solution must be clear, concise, and correct.
- Answers to "yes or no" questions must always be accompanied by a proof or counterexample.

(—) From the textbook: 13.1, 13.2, 13.4 part (a), 13.8

(P1) Let X be a set, and let $\mathscr{T}_\infty = \{U \mid X - U \text{ is infinite or empty or all of } X\}$.
Is $\mathscr{T}_\infty$ a topology on X?

(P2) Show that any open subset of $\mathbb{R}$ (with the standard topology) is a *countable* union of open intervals.

(P3) Consider $\mathbb{R} \times \mathbb{R}$ with the dictionary order. Let us call an interval in this ordered set *vertical* if its endpoints have the same x coordinate. That is, a vertical interval has the form $(a \times b, a \times d)$ for real numbers a, b, d.

Show that the collection of all vertical intervals is a basis for the order topology on $\mathbb{R} \times \mathbb{R}$.

(P4) Suppose that X and Y are discrete topological spaces. Is $X \times Y$ discrete?