Math 320 – Linear Algebra – David Dumas – Fall 2018

Exam 2

Instructions:

- Solve **three** problems from the list below.
- **Justify your answers.**
- The rules for writing proofs are the same as on the most recent homework.
- If a solution involves applying row and/or column operations to a matrix, indicate exactly which operations you are applying at each step.

(1) Consider the following element of $M_{4\times 3}(\mathbb{R})$:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & -2 & 1 \\ 4 & -2 & 6 \\ -2 & 3 & 1 \end{pmatrix}.$$

  (a) Determine the rank of A.
  (b) Find a basis for $R(L_A)$.

(2) The following matrix with entries in the field $\mathbb{Z}_2$ is invertible. Find the inverse matrix.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(3) Consider these three subspaces of $P_2(\mathbb{R})$:

$$W = \{p \mid p(0) = 0 \text{ and } p(1) = 0\}, \quad Y = \{p \mid p(2) = 0\}, \quad Z = \operatorname{span}(x+1).$$

  (a) It is true that $P_2(\mathbb{R}) = W \oplus Y$?
  (b) It is true that $P_2(\mathbb{R}) = W \oplus Z$?

In each part, give a proof of your answer.

(4) Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation defined by $T(p) = x\frac{dp}{dx} - 2p$. Thus for example

$$T(3x^2 + x) = x(6x+1) - 2(3x^2 + x) = -x.$$

  (a) Compute the matrix $[T]_\beta^\beta$ of T with respect to the standard basis $\beta = \{1, x, x^2\}$.
  (b) Let $\gamma = \{x^2 + 2x + 1, 2x + 2, 2\}$, which is also a basis for $P_2(\mathbb{R})$. Find the matrix of T with respect to the basis γ. (That is, find $[T]_\gamma^\gamma$.)