Math 569 / David Dumas / Spring 2017

Presentation topic suggestions

This list was last updated on 2017-02-20.

Notes:

- Topics marked with $*$ are more challenging.
- Students are welcome to suggest a topic that is not on this list.
- Most of the suggestions on this list require some elaboration in order to receive approval. Typically this would involve selecting a primary reference article or textbook.
- If an item on the list below is simply a statement, then the implicit suggestion is to give an exposition of the proof of the statement.

(1) $*$ Every compact surface has a triangulation.

(2) State the classification of noncompact surfaces without boundary as proved by Richards (Trans. AMS, Vol. 106, No. 2 (1963), pp. 259–269). Also give the definitions and discussion necessary for the statement to make sense. Give some examples of homeomorphic and non-homeomorphic infinite-genus surfaces as applications of the theorem.

(3) State and prove the identification between the set of isomorphism classes of vector bundles of rank r over a base X and the set of homotopy classes of maps from X to the infinite Grassmannian $\mathrm{Gr}(n,\infty)$.

(4) $*$ Sketch Penner's hyperbolic-geometric approach to constructing a cell decomposition of the moduli space of hyperbolic surfaces of genus g with n punctures ($n > 0$) in which the cells are in one-to-one correspondence with labeled ribbon graphs of genus g with n boundary components. The original reference is:

  Penner, R. *The decorated Teichmüller space of punctured surfaces*. Comm. Math. Phys. **113** (1987), 299-334.

A recent survey which outlines the proof and discusses it in comparison to other approaches is:

  Norbury, P. *Cell decompositions of moduli space, lattice points, and Hurwitz problems,* in *Handbook of moduli, Vol. 3*. Intl. Press, 2013. arxiv:1006.1153.

(5) Give a short introduction to principal bundles, their relation to vector bundles, criteria for triviality, and some examples such as the frame bundle of a smooth manifold.

(6) Present a proof of Cartan's "Fundamental theorem of calculus" (following Sharpe's book, *Differential Geometry*). This theorem characterizes which forms $\omega \in \Omega^1(M,\mathfrak{g})$ arise as $g^{-1}dg$ for a map $g : M \to G$.

(7) Describe the equivalence between smooth vector bundles over M and finitely generated locally free sheaves over $C^\infty(M)$.

(8) Describe the equivalence between vector bundles over a compact Hausdorff space X and finitely generated projective modules over the ring $C(X)$ of continuous real-valued functions on X.

(9) Connect our discussion of vector bundles and connections with Riemannian geometry: First, define symmetry and torsion for connections on the tangent bundle. Then, prove that there is a unique symmetric torsion-free connection on the tangent bundle that is compatible with any given Riemannian metric.