Math 549: Differentiable Manifolds I – David Dumas – Fall 2017

Homework 12

Due Monday, December 4 at 1:00pm

(1) Let ℓ be a line in $\mathbb{R}^3$. Let $a \in \mathbb{R}^3 - \ell$. Compute the de Rham cohomology of $M = \mathbb{R}^3 - (\ell \cup \{a\})$.

(2) Let ℓ, ℓ' be two lines in $\mathbb{R}^3$ that intersect at a point p. Compute the de Rham cohomology of $M = \mathbb{R}^3 - (\ell \cup \ell')$.

(3) Let $E = \{(t, t^2, t^3) \mid t \geq 0\} \subset \mathbb{R}^3$. Compute the de Rham cohomology of $M = \mathbb{R}^3 - E$.

(4) Recall that $\Omega_c^k(M)$ denotes the space of compactly supported k-forms.
   (a) Show that one can define cohomology spaces
   $$H_c^k(M) := \{\ker d : \Omega_c^k(M) \to \Omega_c^{k+1}(M)\} / \{\operatorname{im} d : \Omega_c^{k-1}(M) \to \Omega_c^k(M)\}$$
   analogously to the de Rham cohomology discussed in lecture. (What needs to be shown here?)
   (b) Show that if M is connected, then $H_c^0(M) = 0$ if and only if M is not compact.
   (c) Compute $H_c^\bullet(\mathbb{R})$.