Math 549: Differentiable Manifolds I – David Dumas – Fall 2017

Homework 1

Due Monday, September 11 at 1:00pm

Notes and instructions

- Write "Homework 1" on the first page of your solutions and label each problem using its number from the list below (*not* the textbook number).
- Tags like [LEE] indicate problems adapted from other sources. The corresponding list of citations can be found on the course web page.

(1) Lee, Chapter 1, Problem 7, on page 30. (This gives another atlas on the sphere.)

(2) Lee, Chapter 1, Problem 9, on page 31. (You will want thoroughly read the construction of a smooth atlas on $\mathbb{RP}^n$ from Examples 1.5 and 1.33 in the chapter before you attempt this problem.)

(3) Let T be the subset of $\mathbb{R}^3$ defined by the equation $(\sqrt{x^2+y^2}-3)^2+z^2=1$. (In words, this is a circular torus of revolution with major radius 3 and minor radius 1.) Construct a smooth atlas on T (of dimension 2). [ND]

(4) Suppose M is a smooth manifold, $p \in M$, and (U,φ) is a coordinate chart such that $p \in U$. Show that for any smooth function $f_0 : \mathbb{R}^n \to \mathbb{R}$, there exists a smooth function $f : M \to \mathbb{R}$ such that f is equal to $f_0 \circ \varphi$ on some open neighborhood of p. [ND]

Here is a suggested outline for solving this problem:

(a) Use a bump function like the one constructed in Lemma 2.22 to construct a smooth function f_1 on $\mathbb{R}^n$ that agrees with f_0 near $\varphi(p)$ but whose support is contained in $\varphi(U)$.

(b) Make f zero on most of the manifold and equal to $f_1 \circ \varphi$ on U. Show that this is smooth.

(5) Lee, Chapter 2, Problem 8, on page 48.

(6) Let M_1, M_2 be smooth manifolds, and for $i \in \{1,2\}$ let $\pi_i : M_1 \times M_2 \to M_i$ denote the projection onto one factor. Let $p = (p_1, p_2) \in M_1 \times M_2$. Show that the map

$$\alpha : T_p(M_1 \times M_2) \to T_{p_1}M_1 \oplus T_{p_2}M_2$$

defined by

$$\alpha(v) = (d(\pi_1)_p(v), d(\pi_2)_p(v))$$

is an isomorphism. [LEE]

(7) Lee, Chapter 3, Problem 8, on page 76.