Practice Midterm Exam

Instructions:

- Complete **three** of the problems below.
- Each problem is worth 10 points.
- If you complete more than three problems (which is *not* recommended) your score will be the sum of your three best problem scores.

Problems:

(1) Consider the circles
$$C_1 = \{z : |z| = 1\}, \ C_2 = \{z : |z - 1| = 3\}.$$
Find a linear fractional transformation T so that $T(C_1)$ and $T(C_2)$ are concentric circles.

(2) Find a conformal mapping from the open first quadrant to the complement of the closed unit disk, i.e. from
$$\{z : 0 < \arg(z) < \frac{\pi}{2}\}$$
to
$$\{z : |z| > 1\}.$$

(3) Let γ be the circle $|z| = 535$ oriented counter-clockwise. Calculate:
$$\int_\gamma \frac{\cos(z/2)}{4z^2 - \pi^2}\, dz$$

(4) Suppose f is a holomorphic function with a zero of order 2 at $z = 0$. Show that there is an analytic function g on an open disk centered at $z = 0$ such that $f(z) = g(z)^2$ on their common domain.

(5) Let f be an entire function, and suppose that f does not take on any positive real values. Show that f is constant.