Math 320 – Linear Algebra – David Dumas – Fall 2016

Final Exam Problems 1 & 2

The final exam will have eight problems. The first two of these problems are shown below. Remember that you cannot bring notes, books, or other written material to the final exam.

(1) Consider the following 3×3 matrix over $\mathbb{R}$ in which one of the entries is an unspecified real number x:

$$A = \begin{pmatrix} 2 & 0 & -3 \\ 0 & 2 & 0 \\ x & 0 & 0 \end{pmatrix}$$

(a) For what values of x does the characteristic polynomial of A split?
(b) For what values of x does the matrix A have three distinct eigenvalues?
(c) For what values of x is the matrix A diagonalizable?

Reminder: In this problem you are working over the field $\mathbb{R}$ of real numbers.

(2) For any positive integer n, let B_n denote the $n \times n$ matrix whose entries are given by

$$(B_n)_{ij} = \begin{cases} 1 & \text{if } |i-j| \leqslant 1 \\ 0 & \text{otherwise.} \end{cases}$$

Thus for example we have

$$B_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \ B_3 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \ B_5 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

For which positive integers n is the matrix B_n is invertible?