Math 320 – Linear Algebra – David Dumas – Fall 2016

Exam 1

Instructions:

- Solve **three** problems from the list below. (If you solve more than three problems, your exam score will be the sum of the three highest problem scores. Since time is limited, I do not recommend attempting more than three problems.)
- Every answer must be justified. Usually that means you need to write a proof.
- You can use any proposition, lemma, or theorem that we covered in lecture thus far or that was in the corresponding sections of the textbook. However, you **must** clearly state what you are using.
- When writing your solutions, clearly label each one with the problem number.
- To receive full credit, a solution must be clear, concise, and correct.

(1) Let $S = \{\,(1,2,-1),\,(3,1,0),\,(0,-5,3)\,\} \subset \mathbb{R}^3$. Consider $\mathbb{R}^3$ as a vector space over $\mathbb{R}$.
  (a) Is S linearly independent?
  (b) Find a subset of S that is a basis for $\operatorname{span}(S)$.

(2) Let $\mathbb{R}[x]_2$ denote the vector space of polynomials of degree at most 2 with real coefficients, which is a vector space over $\mathbb{R}$. Let $T : \mathbb{R}[x]_2 \to \mathbb{R}^3$ be the linear transformation defined by
$$T(p) = (p(1), p(2), p(3)).$$
  (a) Find a basis of the null space of T.
  (b) Find a basis of the range of T.

(3) Let $V = (\mathbb{Z}/2)^4$, a vector space over the field $\mathbb{Z}/2$.
  (a) How many elements does V have?
  (b) Let $W \subset V$ be the set of all vectors that have an even number of entries equal to 1. Show that W is a subspace of V and find a generating set for W.

(4) Suppose V is a vector space and W_1 and W_2 are subspaces of V. Let U be the set of all vectors in V that can be written as a sum of a vector in W_1 and a vector in W_2. That is,
$$U = \{w_1 + w_2 \mid w_1 \in W_1, w_2 \in W_2\}$$
Show that U is a subspace of V.

(5) A square matrix A is called *antisymmetric* if $A^T = -A$. Show that the set of antisymmetric matrices is a subspace of $M_{3\times 3}(\mathbb{R})$ and find a basis for this subspace.