Math 445 – David Dumas – Fall 2015

Midterm Exam

Instructions:

- Solve **three** problems from the list below. (If you solve more than three problems, your exam score will be the sum of the three highest problem scores. Since time is limited, I do not recommend attempting more than three problems.)
- Some problems ask you to write a proof of something that was in the assigned reading, or which we proved in class. In these cases your proof does not need to be the same as the one we discussed, but the proof you give should *not* depend on ideas or results we only covered much later.

(1) (a) [2 points] Write the definition of *continuity* for a function $f : X \to Y$, where X and Y are topological spaces.

(b) [2 points] Write the definition of ε-δ *continuity* for a function $f : X \to Y$, where (X, d_X) and (Y, d_Y) are metric spaces.

(c) [6 points] Show that the topological and ε-δ definitions of continuity are equivalent for maps between metric spaces.

(2) [10 points] Show that a topological space is T_1 if and only if every finite set is closed.

(Recall that a topological space X is said to be T_1 if for every $x, y \in X$ with $x \neq y$ there exists an open set U such that $x \in U$ and $y \notin U$.)

(3) [10 points] Let X be a topological space and $\sim$ an equivalence relation on X. Suppose $X/\sim$ is connected and that each equivalence class of $\sim$ is connected. Show that X is connected.

(4) Consider the set $\mathbb{R}$ with the finite-complement topology. (This is the topology in which the nonempty open sets are exactly the sets with finite complement.) Answer each of the following questions about this topological space, and give a proof of each answer.

(a) [$2\frac{1}{2}$ points] Is it Hausdorff?

(b) [$2\frac{1}{2}$ points] Is it connected?

(c) [$2\frac{1}{2}$ points] Is it path-connected?

(d) [$2\frac{1}{2}$ points] Is it compact?

(5) [10 points] Let X be an ordered set. Show that the order topology on X is Hausdorff.