**Math 445 – David Dumas – Fall 2015**

# Practice Midterm

Instructions:

- Solve **three** problems from the list below. (If you solve more than three problems, your exam score will be the sum of the three highest problem scores. Since time is limited, I do not recommend attempting more than three problems.)
- Some problems ask you to write a proof of something that was in the assigned reading, or which we proved in class. In these cases your proof does not need to be the same as the one we discussed, but the proof you give should *not* depend on ideas or results we only covered much later.

(1) (a) What does it mean for a sequence $x_n$ in a topological space $X$ to *converge to* $x \in X$? (Write the definition.)
(b) Show that if $f : X \to Y$ is continuous, and $x_n \in X$ is a sequence which converges to $x$, then $f(x_n)$ converges to $f(x)$.

(2) Show that a topological space $X$ is Hausdorff if and only if the diagonal
$$\Delta = \{(x,x) \mid x \in X\}$$
is a closed subset of $X \times X$.

(3) Let $X$ be a metrizable topological space. Let $A \subset X$ be a subset and suppose $x \in \bar{A}$. Show that there is a sequence of points $a_n \in A$ such that $a_n$ converges to $x$.

(4) Let $S \subset \mathbb{R}^2$ be the set of all points such that at least one of the coordinates is a rational number. Show that $S$ is connected.

(5) Consider $\mathbb{R}$ with the finite complement topology. Give a simple, explicit description of all of the compact subsets of $X$.