Math 445 – David Dumas – Fall 2015

Homework 6

Due Wednesday, October 7 at 1:00pm

Instructions:

- To receive full credit, a solution must be clear, concise, and correct.
- Problems marked with * are *extra credit problems* and these are *optional*.
- If a problem asks a question with a "yes" or "no" answer, you must provide a proof of whatever answer you give.

(—) From the textbook: 23.1, 23.2, 23.4, 23.5, 23.8, 24.2, 24.7, 24.10

(P1) * A topological space is called *extremely disconnected* if the closure of every open set is open. Give an example of a T_1 topological space that is both connected and extremely disconnected. (Note in particular that this makes the term "extremely disconnected" somewhat misleading.)

(P2) * Let X denote the quotient topological space obtained from $\mathbb{R}$ by collapsing $\mathbb{Q}$ to a point. Let Y denote the quotient of $\mathbb{R}$ by the equivalence relation

$$x \sim y \quad \text{iff} \quad x - y \in \mathbb{Q}.$$

Is X homeomorphic to Y?