Math 445 – David Dumas – Fall 2015

Homework 5

Due Wednesday, September 30 at 1:00pm

Instructions:

- To receive full credit, a solution must be clear, concise, and correct.
- Problems marked with * are *extra credit problems* and these are *optional*.

(—) From the textbook: 20.6, 20.11, 22.2, 22.3, 22.4

(P1) * Let us say that a function $f : X \to Y$ between topological spaces is *sequentially continuous* if $f(x_n) \to f(x)$ whenever $x_n \to x$. Give an example of a function that is sequentially continuous but not continuous. (Include a proof of your claims, not just a description of f, X, and Y.)

(P2) * A sequence x_n is called *eventually constant* if there exist N and x such that $x_n = x$ for all $n \geq N$. Give an example of a topological space in which no singleton set is open, but where a sequence converges if and only if it is eventually constant.

(As a warm-up, you should convince yourself that the problem is much easier if open singleton sets are allowed.)

(P3) * Suppose that (X,d) is a metric space and $f : X \to X$ is a function such that $d(f(x), f(y)) \leq d(x,y)$ for all $x, y \in X$. Show that f is continuous.

(P4) * If (X,d) is a metric space, show that $d : X \times X \to \mathbb{R}$ is continuous, where $X \times X$ has the product of metric topologies and $\mathbb{R}$ has the standard topology.

(P5) * Let $\sim$ denote the equivalence relation on $\mathbb{R} \times \{0,1\}$ generated by $(x,0) \sim (\frac{1}{x}, 1)$ for all $x \neq 0$. Show that the quotient topology on $(\mathbb{R} \times \{0,1\})/\sim$ is homeomorphic to S^1 (the unit circle in the plane with the subspace topology).