Math 445 – David Dumas – Fall 2015

Homework 4

Due Monday, September 21 at 1:00pm

Instructions:

- Problems from the textbook (Munkres, 2ed) are specified in the format "Section.Exercise".
- To receive full credit, a solution must be clear, concise, and correct.
- If a problem asks a question with a "yes" or "no" answer, you must provide a proof of whatever answer you give.

(—) From the textbook: 18.9, 19.7, 20.2, 20.3, 20.4

(P1) Suppose X is a set and $\mathscr{T}, \mathscr{T}'$ are topologies on X. In each part of this problem, determine whether or not the given condition implies that $\mathscr{T} = \mathscr{T}'$. If so, give a proof. If not, give a counterexample and prove that it is a counterexample.

(a) For every finite topological space W, a function $f : W \to X$ is continuous with respect to $\mathscr{T}$ if and only if it is continuous with respect to $\mathscr{T}'$.

(b) For every finite topological space Y, a function $f : X \to Y$ is continuous with respect to $\mathscr{T}$ if and only if it is continuous with respect to $\mathscr{T}'$.

(c) For every Hausdorff topological space Y, a function $f : X \to Y$ is continuous with respect to $\mathscr{T}$ if and only if it is continuous with respect to $\mathscr{T}'$.

(P2) Let $C([0,1])$ denote the set of all continuous real-valued functions on the interval $[0,1]$. Given elements $f, g \in C([0,1])$, define

$$d_\infty(f,g) = \sup_{x \in [0,1]} |f(x) - g(x)|.$$

and

$$d_1(f,g) = \int_0^1 |f(x) - g(x)| dx.$$

(a) Show that d_∞ is a metric on $C([0,1])$.

(b) Show that d_1 is a metric on $C([0,1])$.

(c) Do d_∞ and d_1 define the same topology on $C([0,1])$?