Math 445 – David Dumas – Fall 2015

Homework 3

Due Monday, September 14 at 1:00pm

Instructions:

- Problems from the textbook (Munkres, 2ed) are specified in the format "Section.Exercise".
- When writing your solutions, clearly label each one with the problem number. Use the same numbering as in the homework sheet (i.e. 13.8, P2, etc.)
- To receive full credit, a solution must be clear, concise, and correct.
- Problems marked with * are *challenge problems* and these are *optional*. Correct solutions to these problems will earn a small amount of extra credit.

(—) From the textbook: 17.6, 17.13, 18.2, 18.3, 18.5, 18.7a

For the problems below, consider the set X of continuous real-valued functions on the interval $[0,1]$. Given a function $f \in X$ we can define two subsets of X as follows:

$$U_f = \{g \in X \mid f(x) < g(x) \text{ for all } x \in [0,1]\}$$
$$U^f = \{g \in X \mid g(x) < f(x) \text{ for all } x \in [0,1]\}$$

Let $\mathscr{C}$ denote the collection of all sets of this form, i.e. $\mathscr{C} = \{U_f \mid f \in X\} \cup \{U^f \mid f \in X\}$.

(P1) Show that $\mathscr{C}$ is a subbasis on X.

For all subsequent problems we consider X to be a topological space with the topology generated by $\mathscr{C}$.

(P2) Show that X is Hausdorff.

(P3) Suppose that a sequence f_n in X converges to f. Show that for any $x \in [0,1]$, the sequence of real numbers $f_n(x)$ converges to $f(x)$.

(P4) * Show that the converse of the statement in the previous problem is false. That is, exhibit a sequence $f_n \in X$ such that for each $x \in [0,1]$ the sequence of real number $f_n(x)$ converges, but the sequence f_n does not converge.

(P5) * Give a *countable* basis for the topology of X.