Math 445 – David Dumas – Fall 2015

Homework 2

Due Wednesday, September 9 at 1:00pm

Instructions:

- Problems from the textbook (Munkres, 2ed) are specified in the format "Section.Exercise".
- When writing your solutions, clearly label each one with the problem number. Use the same numbering as in the homework sheet (i.e. 13.8, P2, etc.)
- To receive full credit, a solution must be clear, concise, and correct.

Note: When a problem asks a question with a "yes" or "no" answer, you are expected to provide a proof of whatever answer you give!

(—) From the textbook: 13.8, 16.2, 16.3, 16.8, 16.9, 17.1, 17.3, 17.5

(P1) The rationals $\mathbb{Q}$ are a subset of the ordered set of real numbers $\mathbb{R}$. Is the subspace topology on $\mathbb{Q}$ the same as its order topology?

(P2) Can you realize the dictionary order topology of $X = \{1,2\} \times \mathbb{Z}^+$ as the subspace topology of some subset of $\mathbb{R}$? That is, can you find a set $Y \subset \mathbb{R}$ and a bijection $f : X \to Y$ such that $U \subset X$ is open in the dictionary order topology if and only if $f(U) \subset Y$ is open in the subspace topology?

(P3) Let X be a topological space, $\mathscr{P}(X)$ the power set of X, and $Cl : \mathscr{P}(X) \to \mathscr{P}(X)$ the "closure operator" defined by $Cl(A) = \bar{A}$. Show that the map C has the following properties. (Some of these will be very easy to prove!)

(a) $Cl(\emptyset) = \emptyset$
(b) $Cl(A \cup B) = Cl(A) \cup Cl(B)$ for all $A, B \in \mathscr{P}(X)$
(c) $Cl(Cl(A)) = Cl(A)$ for all $A \in \mathscr{P}(X)$
(d) $A \subset Cl(A)$ for all $A \in \mathscr{P}(X)$

(P4) Let X be a set, and let $Cl : \mathscr{P}(X) \to \mathscr{P}(X)$ be a map that has properties (a)-(d) listed in the previous problem. Define

$$\mathscr{T} = \{U \in \mathscr{P}(X) \,|\, Cl(X \setminus U) = X \setminus U\}.$$

Show that $\mathscr{T}$ is a topology on X and that Cl is the closure operator of this topology.