

## Homework 11

Due Wednesday, November 25 at 1:00pm

Instructions:

- To receive full credit, a solution must be clear, concise, and correct.

(—) From the textbook: 43.4, 43.5, 45.2, 45.4a

A topological  $X$  space is called *hemicompact* if there is a countable collection  $\{K_i\}_{i \in \mathbb{N}}$  of compact subsets of  $X$  such that for every compact set  $K \subset X$  there exists  $n$  with  $K \subset K_n$ .

- (P1) Show that if  $X$  is hemicompact and  $Y$  is a metric space, then the topology of compact convergence on  $C(X, Y)$  is metrizable. (Hint: The basic idea is to sum the sup metrics on  $C(K_n, Y)$  for all  $n$ , but without some modification the sum may not converge.)
- (P2) Show that if  $X$  is hemicompact, then  $X$  has a countable cover by compact sets.
- (P3) Show that a hemicompact and first countable space is locally compact.
- (P4) Give an example of a space that has a countable cover by compact sets but is not hemicompact. (Hint: use the previous problem.)