Math 445 – David Dumas – Fall 2015

Homework 1

Due Monday, August 31 at 1:00pm

Instructions:

- Problems from the textbook (Munkres, 2ed) are specified in the format "Section.Exercise". For example, "13.8" means section 13, exercise 8, which is the last problem on page 83.
- When writing your solutions, clearly label each one with the problem number. Use the same conventions as in the homework sheet (i.e. 13.8, 14.2, etc., for book problems, P1, P2, etc. for non-book problems).
- To receive full credit, a solution must be clear, concise, and correct.
- Problems marked with * are *challenge problems* and these are *optional*. Correct solutions to these problems will earn a small amount of extra credit.

(—) From the textbook: 13.1, 13.2, 13.3, 13.4, 13.5

(P1) Show that any open subset of $\mathbb{R}$ (with the standard topology) is a *countable* union of open intervals.

(P2) Define a *rational open ray* to be a subset of $\mathbb{R}$ of the form (a,∞) or $(-\infty,a)$ where a is a rational number. Show that the collection $\mathscr{S}$ of all rational open rays is a subbasis for the standard topology on $\mathbb{R}$.

(P3) If X and Y are topological spaces, a function $f : X \to Y$ is called *continuous* if for every open set $U \subset Y$, the preimage $f^{-1}(U)$ is open in X.

(a) Let $\mathbb{R}_d$ denote the set of real numbers with the discrete topology. Show that every function $f : \mathbb{R}_d \to \mathbb{R}$ is continuous.

(b) Let $\mathbb{R}_{triv}$ denote the set of real numbers with the trivial topology. Show that every continuous function $f : \mathbb{R}_{triv} \to \mathbb{R}$ is constant.

(c) Let $\mathbb{R}_{cc}$ denote the set of real numbers with the *countable complement topology*. This is the topology in which a set U is open if it is empty or if $\mathbb{R} \setminus U$ is countable. Is every continuous function $f : \mathbb{R}_{cc} \to \mathbb{R}$ constant? (If yes, prove it. If no, give an example of a nonconstant continuous function of this type and a proof that your example is continuous.)

(P4) * Show that every continuous function $f : \mathbb{R} \to \mathbb{R}_d$ is constant.