MCS 481 - Computational Geometry - David Dumas

Homework 7

Due Monday, April 21

Note: Most of the problems assigned from the textbook are about geometric duality, which we will discuss starting on April 14.

From the textbook.

8.2, 8.4, 8.11

Other problems.

Recall that a *Davenport-Schinzel sequence of order s on n symbols* is a sequence S of integers from $\{1, \ldots, n\}$ such that

- Neighboring elements of S are distinct ("no repeats")
- S contains no alternating subsequence of the form $\ldots a \ldots b \ldots a \ldots b \ldots$ of total length $s + 2$

The set of all such sequences is denoted $DS(n, s)$ and the maximum length of such a sequence is denoted $\lambda_s(n)$.

For example, we have $12321 \in DS(3, 2)$ but $12332 \notin DS(3, 2)$ because it has a repeated symbol, and $123213 \notin DS(3, 2)$ because it contains $1 \ldots 3 \ldots 1 \ldots 3$.

The problems below ask you to prove some things about Davenport-Schinzel sequences. In case you get stuck and refer to a textbook for help, be sure to cite it and also explain the answer in your own words (reflecting your understanding of the material, not simply copying).

(P1) Show that for all s and n, the maximum length $\lambda_s(n)$ is finite.

(P2) In this problem you will prove that $\lambda_3(n)$, the maximum length of a Davenport-Schinzel sequence of order 3, is $O(n \log n)$. Note that in $DS(n, 3)$ the excluded alternating pattern is *ababa*.

(a) Let $S \in DS(n, 3)$ have length k. Show that there exists a sequence $S' \in DS(n - 1, 3)$ of length at least

$$k - \left\lfloor \frac{k}{n} \right\rfloor - 2.$$

Hint: Delete the least-often-used symbol from S. Now the sequence my have repeated characters. But how many?

(b) Write the result of (a) as an inequality satisfied by $\lambda_3(n)$ and $\lambda_3(n - 1)$.

(c) Solve this recurrence relation for a bound on $\lambda_3(n)$ that is $O(n \log n)$.