Math 180 / Spring 2014 / David Dumas

Quiz 6 Solution

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Problem. Suppose y is a function of x that satisfies the equation

$$x^2 - 2y = y^2 - 2x.$$

Find $\frac{dy}{dx}$ at $(x, y) = (-4, 2)$.

Solution 1. Using implicit differentiation of the equation we find

$$2x - 2\frac{dy}{dx} = 2y\frac{dy}{dx} - 2$$

or, after a bit of algebra,

$$(2y + 2)\frac{dy}{dx} = 2x + 2$$

which we solve for $\frac{dy}{dx}$ to obtain

$$\frac{dy}{dx} = \frac{2x + 2}{2y + 2} = \frac{x + 1}{y + 1}.$$

Substituting $(-4, 2)$ gives

$$\left.\frac{dy}{dx}\right|_{(x,y)=(-4,2)} = \frac{-4 + 1}{2 + 1} = \boxed{\text{-1}}.$$

Solution 2. Instead of using implicit differentiation, in this case one can also solve for y as a function of x directly.

First, moving all x and y terms to opposite sides we find

$$y^2 + 2y = x^2 + 2x.$$

Adding 1 to both sides completes the square, yielding

$$(y + 1)^2 = y^2 + 2y + 1 = x^2 + 2x + 1 = (x + 1)^2.$$

This says $(y+1)$ and $(x+1)$ are real numbers with the same square, so they are either equal or differ by a sign:

$$y + 1 = \pm(x + 1).$$

Rearranging, we find that he two solution functions are

$$y = (x + 1) - 1 = x$$

and

$$y = -(x + 1) - 1 = -x - 2.$$

Notice that the point $(-4, 2)$ does *not* satisfy $y = x$, so it must lie on the graph of the second solution, $y = -x - 2$.

Differentiating $y = -x - 2$ we find $\frac{dy}{dx} = \boxed{\text{-1}}$.