Math 180 / Spring 2014 / David Dumas

Quiz 5 Solution

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Problem. The parabola $y = \frac{1}{2}x^2 - x + 2$ has two tangent lines that pass through $(0,0)$. Find the equation of one of these lines.

Solution. The parabola is the graph of the function $f(x) = \frac{1}{2}x^2 - x + 2$ which has derivative $f'(x) = x - 1$.

The tangent line to the curve $y = f(x)$ at $x = a$ has equation:

$$y - f(a) = f'(a) \cdot (x - a)$$

Substituting the formulas for $f(a)$ and $f'(a)$ in this case we have

$$y - (\frac{1}{2}a^2 - a + 2) = (a - 1)(x - a).$$

Putting this linear equation in slope-intercept form we find

$$y = (a - 1)x + (2 - \frac{1}{2}a^2).$$

This tangent line passes through $(0,0)$ exactly when the y-intercept is zero, i.e. when

$$2 - \frac{1}{2}a^2 = 0$$

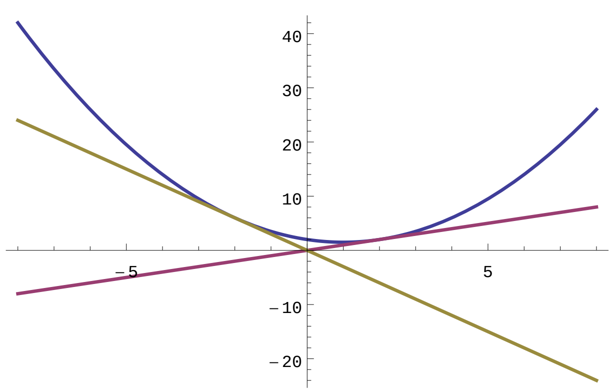
which has two solutions, $a = 2$ and $a = -2$. The slopes of these lines are $f'(2) = 1$ and $f'(-2) = -3$, so the equations of the tangent lines are

$$\boxed{\text{y = x}}$$

and

$$\boxed{\text{y = -3x.}}$$

Graph. The parabola and the two tangents found above are shown in the figure below.



More help with tangents. Review the definition of the tangent line and its slope in section 3.1 of the textbook. Then look at examples 4 and 5 in section 3.2.