Math 180 / Spring 2014 / David Dumas

Quiz 5: Common mistake #1

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Problem. The parabola $y = \frac{1}{2}x^2 - x + 2$ has two tangent lines that pass through $(0, 0)$. Find the equation of one of these lines.

Common mistake. By far the most common incorrect answer given by students taking this quiz was

$$y = -x$$

which was typically derived by the following process:

(1) Find the slope of the tangent line to $y = \frac{1}{2}x^2 - x + 2$ at $x = 0$.
(2) Find the equation of a line with this slope passing through $(0, 0)$.

There are two problems with this approach.

The first is that the tangent line at $x = 0$ does not pass through $(0, 0)$, so it is not one of the lines the problem asks about. In fact, since $(0, 2)$ is the point on this parabola with $x = 0$, and $dy/dx = -1$ at $x = 0$, the equation of this tangent line is

$$y - 2 = -1(x - 0)$$

or, in slope-intercept form

$$y = -x + 2.$$

The nonzero y-intercept shows this line does not pass through the origin.

The second problem is that in step (2) one is changing the y-intercept to zero, so the line is no longer a tangent of the parabola. Here the effect is to move the tangent line down two units so that is passes through the origin.

Graph. The figure below shows the tangent line to the parabola at $x = 0$ (dotted line), and the parallel line $y = -x$ which passes through $(0, 0)$ (solid line).

Notice that the dotted line is a tangent but does not pass through $(0, 0)$, while the solid line passes through $(0, 0)$ but is not a tangent. The problem asks for a line with both properties. (One way to find such a line is explained in the quiz 5 solution.)