Math 550 - Differentiable Manifolds II

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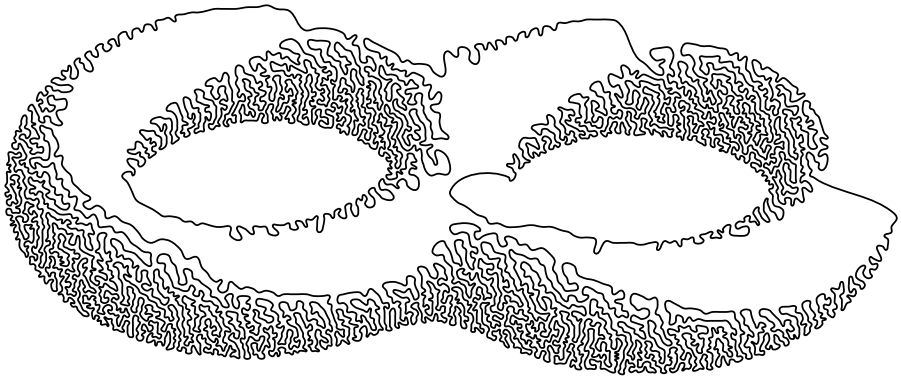
Fall 2014



1. Basic course information

Web Page	http://www.math.uic.edu/~ddumas/math550/
Location	Taft Hall 309
Meeting Time	MWF 10am
CRN	37063
Email	ddumas@math.uic.edu
Office Hours	Monday and Friday 2-3pm

2. Texts

The lectures will draw from a number of resources, primarily the texts listed below. You are not required to purchase any of these. Relevant chapters or sections for each component of the course will be announced in lecture.

- M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Volumes 1, 2, and 3. (Publish or Perish, 1999)
- M. Lee, *Introduction to Smooth Manifolds*. (Springer GTM, 2012)
- F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*. (Springer GTM, 1983)
- R. W. Sharpe, *Differential Geometry: Cartan's Generalization of Klein's Erlangen Program*. (Springer GTM, 2000)
- A. Cannas da Silva, *Lectures on Symplectic Geometry*. (Springer LNM, 2006) Online version: http://www.math.ist.utl.pt/~acannas/Books/lsg.pdf
- D. McDuff and D. Salamon, *Introduction to Symplectic Topology*. (Oxford, 1999)

3. Course overview

Building on the foundational material from Math 549, we will discuss several aspects of the geometry and topology of smooth manifolds. Topics will include de Rham theory, distributions and foliations, Lie groups, Lie algebras, vector bundles, principal bundles, and symplectic geometry.

4. Prerequisites

- Math 549 or equivalent experience, and
- Some basic homological algebra (e.g. chain complexes, exact sequences, homology and cohomology).

5. Topic outline

The topics may be adjusted slightly to account for time constraints and for the background and preferences of the students.

(1) De Rham theory
  (a) De Rham cohomology groups
  (b) The Mayer-Vietoris sequence
  (c) The Poincáre lemma
  (d) Triangulated manifolds and the Euler characteristic
  (e) De Rham's theorem
  (f) Index of a vector field
(2) Lie groups and Lie algebras
  (a) Definitions and examples
  (b) Left-invariant vector fields and parallelization
  (c) Cohomology of a compact Lie group
  (d) Frobenius theorem and distributions
  (e) The Maurer-Cartan form
  (f) Maps into a Lie group and the Darboux derivative
  (g) Integrability of 1-forms with values in a Lie algebra
  (h) Applications to curves and surfaces
(3) Bundles
  (a) Fiber bundle generalities
  (b) Vector bundles
  (c) Functorial constructions (tensor, sum, dual)
  (d) Principal bundles
  (e) Associated bundles
  (f) Connections on principal bundles
  (g) Connections on vector bundles
  (h) Curvature and parallel transport
  (i) Reduction of structure group
  (j) Examples and applications
(4) Symplectic geometry
  (a) Symplectic vector spaces
  (b) Symplectic manifolds
  (c) Obstructions to existence of a symplectic structure
  (d) Cotangent bundles
  (e) Lagrangian submanifolds
  (f) Darboux theorem
  (g) Hamiltonian flows
  (h) Poisson bracket
  (i) Poisson manifolds
  (j) Connection with classical mechanics

6. Course policies

Students are expected to attend all of the lectures.

There are no exams.

A list of problems will be maintained on the course web page and expanded throughout the semester. Any exercises mentioned in lecture will be added to this list, but the list will also contain additional problems. Some of the problems will be marked with an asterisk (*); these *challenge problems* are considered to be more difficult or open-ended.

I would like each student to submit written solutions to a few problems from the list each week. However, rather than making weekly problem submission a strict requirement, I will enforce the somewhat more flexible policy described below.

Students are required to submit problems from the list on a regular basis. Course grades will be based on the problems received from each student. Any number of problems may be submitted at any time, however:

- Each student must submit at least **five** problems during each of the three-week periods in the table below, and
- At least **one** of the problems submitted in each period must be a challenge problem.

Period	End date
1	Fri, Sep 12
2	Fri, Oct 3
3	Fri, Oct 24
4	Fri, Nov 14
5	Fri, Dec 5

Problems will be held to a high standard for both mathematical correctness and clarity.

Resubmission of problems (for the chance at a higher score) is allowed during periods 1–4, i.e. before November 14. All previously submitted versions of a problem must be attached when resubmitting. Resubmissions do not count toward the minimum number of problems you must submit in each period.

Students are encouraged to discuss course material and problems with one another. However, collaboration must be acknowledged in any work that is submitted for credit (for example by writing "In collaboration with Jane Doe" at the start of the solution to a problem).

7. Academic Honesty

All UIC students (graduate and undergraduate) are required to maintain the standards of academic integrity described in the *Guidelines Regarding Academic Integrity*:

http://www.uic.edu/ucat/catalog/GR.shtml#qa

In particular, this policy prohibits plagiarism. Any violation of these standards will be handled in accordance with the Student Disciplinary Policy.