Math 180 / David Dumas / Fall 2014

Solution and Rubric for Quiz 9 (Wed Oct 29)

Problem: Give an example of a function that has all of the following properties:

- $f(x)$ is defined for all x
- $f(x)$ is differentiable for all x
- $f(5) = -7$
- $x = 5$ is the only critical point of f
- $x = 5$ is a local maximum and absolute maximum of f

Solution: The problem is open-ended and has many solutions. Here is one of them. Using a polynomial function will ensure it is defined and differentiable for all x. We want a function with only one critical point. A prototype for this kind function is the parabola

$$f(x) = x^2$$

which has its only critical point at $x = 0$, which is a local and absolute minimum. If we flip it over,

$$f(x) = -x^2$$

then we still have just one critical point, but now it is a local and absolute maximum. But the critical point is still at $x = 0$. To move this over to $x = 5$ we replace x by $x - 5$ in the formula above, i.e.

$$f(x) = -(x-5)^2$$

Now this satisfies everything except $f(5) = -7$. Instead the function above has $f(5) = 0$. If we add a constant to the function, the derivative and therefore the critical points do not change, nor does the location of an absolute maximum. So adding -7 we finally get a function with all of the required properties:

$$\boxed{f(x) = -7 - (x-5)^2}$$

Rubric:

- If the final answer is correct, and is supported by clear and correct work: $\boxed{\text{2 points}}$
- If there is significant progress toward finding a function with at least one of the given properties other than $f(5) = -7$, including but not limited to
  - A clear final answer is given, and is a quadratic polynomial with explicit coefficients, OR
  - A graph of a function with all of the required properties is shown, with some attempt to turn the graph into a formula for $f(x)$,

  then: $\boxed{\text{1 point}}$
- Otherwise: $\boxed{\text{0 points}}$