Math 180 / David Dumas / Fall 2014

Solution and Rubric for Quiz 7 (Fri Oct 17)

Problem: Find the tangent line to $y = \cot^{-1}(x)$ at $x = \sqrt{3}$. Write your answer in slope-intercept form (y = mx + b).

Solution: The derivative is

$$\frac{dy}{dx} = \frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$$

so the slope of the tangent line is

$$m = \frac{dy}{dx}\Big|_{x=\sqrt{3}} = \frac{-1}{1+3} = -\frac{1}{4}.$$

Since $\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$, the equation of the tangent line is

$$x - \frac{\pi}{6} = -\frac{1}{4}(x - \sqrt{3}).$$

The slope-intercept form of this equation is:

$$y = -\frac{1}{4}x + \left(\frac{\sqrt{3}}{4} + \frac{\pi}{6}\right)$$

Rubric:

- If the correct final answer is given in slope-intercept form, and is fully supported by clear and correct work (including a formula for the derivative of $\cot^{-1}(x)$): 2 points
- Otherwise, if at least one of the following was computed correctly,

• Slope $m = -\frac{1}{4}$ • $\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$ then: 1 point

• Otherwise: 0 points