Math 180 / David Dumas / Fall 2014

Solution and Rubric for Quiz 15 (Wed Dec 3)

Problem: Compute $\dfrac{d}{dx}\left(\displaystyle\int_1^{\frac{1}{x}} \frac{1}{\ln t}\,dt\right)$.

Solution: Let F be any antiderivative of $\frac{1}{\ln x}$. Then we have

$$\int_1^{\frac{1}{x}} \frac{1}{\ln t}\,dt = F(\tfrac{1}{x}) - F(1)$$

Taking the derivative, and using the chain rule, we find

$$\begin{aligned}\frac{d}{dx}\left(\int_1^{\frac{1}{x}} \frac{1}{\ln t}\,dt\right) &= \frac{d}{dx}F(\tfrac{1}{x}) - \frac{d}{dx}F(1) \\ &= F'(\tfrac{1}{x}) \cdot \left(\frac{-1}{x^2}\right) - 0.\end{aligned}$$

Since F is an antiderivative of $\frac{1}{\ln x}$, we have $F'(\frac{1}{x}) = \frac{1}{\ln(\frac{1}{x})}$. Therefore

$$\frac{d}{dx}\left(\int_1^{\frac{1}{x}} \frac{1}{\ln t}\,dt\right) = \frac{-1}{x^2 \ln(\frac{1}{x})}.$$

Correction: There is a small problem with the quiz question as stated. The lower limit of integration is 1, which is not in the domain of $\frac{1}{\ln t}$. However, we see above that the constant lower limit of integration plays no role in the calculation. Still, the problem should have been stated with a different lower limit, such as

$$\frac{d}{dx}\left(\int_2^{\frac{1}{x}} \frac{1}{\ln t}\,dt\right).$$

Anyone who pointed out this issue in their answer received full credit.

Rubric:

- If the final answer is correct and is supported by clear and correct work: $\boxed{\text{1 point}}$
- If the fact that the lower limit of integration is not in the domain of the integrand is observed: $\boxed{\text{1 point}}$
- Otherwise: $\boxed{\text{0 points}}$