

## Solution and Rubric for Quiz 12 (Wed Nov 5)

**Problem:** Use linear approximation to estimate  $\sqrt{80}$ . Write your answer as a single fraction  $p/q$ , where  $p$  and  $q$  are integers.

**Solution:** We notice that 80 is close to the perfect square  $81 = 9^2$ , so  $\sqrt{80}$  will be close to 9. To get a better estimate we compute the linear approximation to  $f(x) = \sqrt{x}$  at the point  $x = 81$ .

In general, the linear approximation of  $f$  at  $x = a$  is

$$L(x) = f(a) + f'(a) \cdot (x - a).$$

Since  $f(81) = 9$  and  $f'(81) = \frac{1}{2\sqrt{81}} = \frac{1}{18}$ , in this case we have

$$L(x) = f(81) + f'(81)(x - 81) = 9 + \frac{1}{18}(x - 81)$$

The linear approximation of  $\sqrt{80}$  is therefore

$$f(x) \approx L(80) = 9 + \frac{1}{18}(80 - 81) = 9 - \frac{1}{18} = \frac{161}{18}.$$

**Rubric:**

- If the final answer is correct, and is supported by clear and correct work: 2 points
- Otherwise, if the function  $f(x) = \sqrt{x}$  and  $a = 81$  were recognized and the linear function  $L(x)$  was computed correctly: 1 point
- Otherwise: 0 points