Solution and Rubric for Quiz 1 (Fri Sep 5)

Problem: Let $f(x) = \dfrac{3x}{5x^3 - 4x}$.

   (a) Compute $f(0)$.

   (b) Compute $\lim_{x \to 0} f(x)$.

Solution for (a):

When $x = 0$, the denominator is $(5 \cdot 0^3 - 4 \cdot 0) = 0$, so $x = 0$ is not in the domain of the function. Therefore,

$$\boxed{\text{The function } f \text{ is not defined at } x = 0.}$$

Rubric for (a):

- If the correct answer is the only answer given, and if it is fully supported by clear and correct work: $\boxed{2 \text{ points}}$
- Otherwise: $\boxed{0 \text{ points}}$

Common mistakes and notes for (a):

- Writing $0/0$ is not enough. Because $0/0$ does not represent a number, it is necessary to indicate what this *means*, which is that $x = 0$ is not in the domain of the function.
- Part (a) does not involve limits at all, so writing "lim" anywhere in the solution of this part is an error.
- No partial credit is available on this part of the problem.

Solution to (b):

Factoring and canceling we find that for $x \neq 0$ this function is equal to another rational function whose denominator has nonzero limit, so we can apply the quotient rule:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{3x}{x(5x^2 - 4)} = \lim_{x \to 0} \frac{3}{5x^2 - 4} = \boxed{\frac{-3}{4}}$$

Rubric for (b):

- If the correct answer is the only answer given, and if it is fully supported by clear and correct work: $\boxed{3 \text{ points}}$
- Otherwise: $\boxed{0 \text{ points}}$

Common mistakes and notes for (b):

- A common error in limit problems like this one is failing to write limit signs in some expressions, such as:

$$\lim_{x \to 0} \frac{3x}{x(5x^2 - 4)} = \frac{3}{5x^2 - 4} \quad \leftarrow \text{INCORRECT!}$$

- Because this is a straightforward factor-and-cancel problem with particularly simple algebra, no partial credit was given. Any mistake (such as a missing sign, or an arithmetic error) results in no credit for this part.

A previous version of this rubric contained a typographical error. This updated version was posted on Monday, September 8